

3.6 Fourier Analysis MATLAB Laboratory Experiment

Purpose: This experiment demonstrates approximations of periodic signals by truncated Fourier series as defined in formula (3.4). Using MATLAB students will plot the actual approximate signals and observe, for large values of N , the Gibbs phenomenon at the jump discontinuity points. In addition, students will use MATLAB to plot the system frequency spectra, and to find the system response due to periodic inputs.

Part 1. Find the trigonometric form of the Fourier series for the periodic signal presented in Figure 3.18. Take $E = 1$, $T = 1$ and use MATLAB to calculate the coefficients of the Fourier series for $n = 0, 1, 2, \dots, N$. Plot the approximations $x_N(t)$ as defined in (3.4) for $N = 5, 10, 20, 30, 40, 50$. Observe the Gibbs phenomenon and for $N = 50$ estimate the relative magnitude of ripples at the jump discontinuity points.

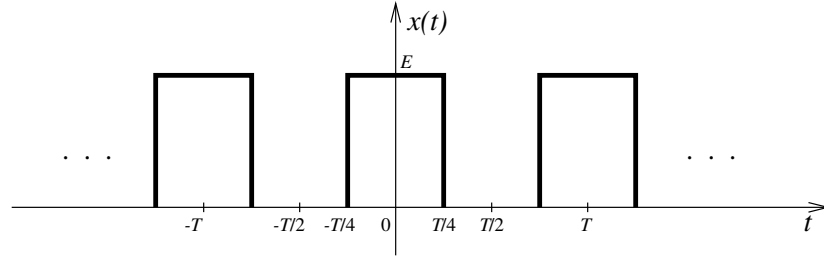


FIGURE 3.18: A square wave signal

Part 2. Use MATLAB to plot the amplitude and phase line spectra of the periodic signal from Part 1.

Part 3. Plot the magnitude and phase spectra of the system defined in Example 3.19 by using the MATLAB function `freqs(num,den)`, where the vectors `num` and `den` contain the coefficients of the transfer function numerator and denominator in descending order.

Part 4. For the system defined in Example 3.19, and $x_N(t)$ determined in Part 1 with $E = 5$, $\omega_0 = 1 \text{ rad/s}$, calculate the Fourier series coefficients of the output signal for $n = 0, 1, 3, 5$. Print the values for the magnitudes of the Fourier series coefficients of the output signal. Plot the approximations $x_N(t)$ and observe the convergence of the output signal as N increases. Take $N = 1, 3, 5$ and $t \in [0, 4\pi]$. Comment on the frequency of the output signal and check its value from the plot obtained.

Submit all plots and comment on the results obtained.

SUPPLEMENT:

$$x_N(t) = \frac{1}{2}a_0 + \sum_{n=1}^N [a_n \cos(n\omega_0 t) + b_n \sin(n\omega_0 t)] \quad (3.4)$$

The system in Example 3.19 is defined by

$$\frac{d^2 y(t)}{dt^2} + 2\frac{dy(t)}{dt} + 3y(t) = x(t)$$