1.9 Exercises

1. Evaluate the following functions:

a. $\sin t\delta\left(t-\frac{\pi}{6}\right)$

b. $\cos 2t\delta\left(t-\frac{\pi}{4}\right)$

c. $\cos^2 t\delta\left(t-\frac{\pi}{2}\right)$

d. $\tan 2t\delta\left(t-\frac{\pi}{8}\right)$

e. $\int_{-\infty}^{\infty} t^2 e^{-t}\delta(t-2)dt$

f. $\sin^2 t\delta^1\left(t-\frac{\pi}{2}\right)$

2.

a. Express the voltage waveform $v(t)$ shown in Figure 1.24, as a sum of unit step functions for the time interval $0 < t < 7\ s$.

b. Using the result of part (a), compute the derivative of $v(t)$, and sketch its waveform.

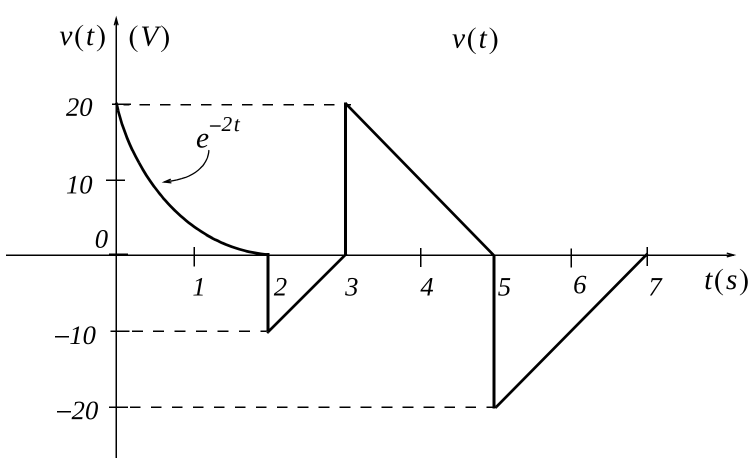

Figure 1.24. Waveform for Exercise 2

2.6 Exercises

1. Find the Laplace transform of the following time domain functions:

   a. 12

   b. $6u_0(t)$

   c. $24u_0(t-12)$

   d. $5tu_0(t)$

   e. $4t^5u_0(t)$

2. Find the Laplace transform of the following time domain functions:

   a. $j8$

   b. $j5\angle -90°$

   c. $5e^{-5t}u_0(t)$

   d. $8t^7e^{-5t}u_0(t)$

   e. $15\delta(t-4)$

3. Find the Laplace transform of the following time domain functions:

   a. $(t^3+3t^2+4t+3)u_0(t)$

   b. $3(2t-3)\delta(t-3)$

   c. $(3\sin 5t)u_0(t)$

   d. $(5\cos 3t)u_0(t)$

   e. $(2\tan 4t)u_0(t)$ Be careful with this! Comment and skip derivation.

4. Find the Laplace transform of the following time domain functions:

   a. $3t(\sin 5t)u_0(t)$

   b. $2t^2(\cos 3t)u_0(t)$

   c. $2e^{-5t}\sin 5t$

d. $8e^{-3t}cos4t$

e. $(cost)\delta(t-\pi/4)$

5. Find the Laplace transform of the following time domain functions:

a. $5tu_0(t-3)$

b. $(2t^2-5t+4)u_0(t-3)$

c. $(t-3)e^{-2t}u_0(t-2)$

d. $(2t-4)e^{2(t-2)}u_0(t-3)$

e. $4te^{-3t}(cos2t)u_0(t)$

6. Find the Laplace transform of the following time domain functions:

a. $\frac{d}{dt}(sin3t)$

b. $\frac{d}{dt}(3e^{-4t})$

c. $\frac{d}{dt}(t^2cos2t)$

d. $\frac{d}{dt}(e^{-2t}sin2t)$

e. $\frac{d}{dt}(t^2e^{-2t})$

7. Find the Laplace transform of the following time domain functions:

a. $\frac{sint}{t}$

b. $\int_0^t \frac{sin\tau}{\tau}d\tau$

c. $\frac{sinat}{t}$

d. $\int_t^{\infty} \frac{cos\tau}{\tau}d\tau$

e. $\int_t^{\infty} \frac{e^{-\tau}}{\tau} d\tau$

8. Find the Laplace transform for the sawtooth waveform $f_{ST}(t)$ of Figure 2.8.

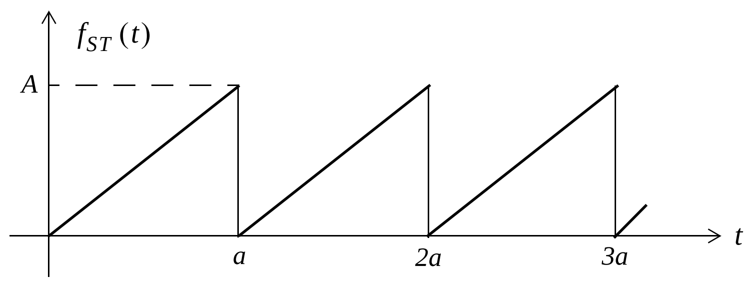

Figure 2.8. Waveform for Exercise 8.

9. Find the Laplace transform for the full rectification waveform $f_{FR}(t)$ of Figure 2.9.

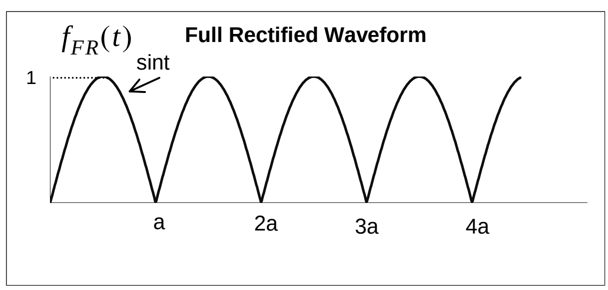

Figure 2.9. Waveform for Exercise 9

3.6 Exercises

1. Find the Inverse Laplace transform of the following:

a. $\dfrac{4}{s+3}$

b. $\dfrac{4}{(s+3)^2}$

c. $\dfrac{4}{(s+3)^4}$

d. $\dfrac{3s+4}{(s+3)^5}$

e. $\dfrac{s^2+6s+3}{(s+3)^5}$

2. Find the Inverse Laplace transform of the following:

a. $\dfrac{3s+4}{s^2+4s+85}$

b. $\dfrac{4s+5}{s^2+5s+18.5}$

c. $\dfrac{s^2+3s+2}{s^3+5s^2+10.5s+9}$

d. $\dfrac{s^2-16}{s^3+8s^2+24s+32}$

e. $\dfrac{s+1}{s^3+6s^2+11s+6}$

3. Find the Inverse Laplace transform of the following:

a. $\dfrac{3s+2}{s^2+25}$

b. $\dfrac{5s^2+3}{(s^2+4)^2}$ (See hint on next page)

Hint: $\left\{\begin{array}{l} \frac{1}{2\alpha}(\sin\alpha t + \alpha t\cos\alpha t) \Leftrightarrow \frac{s^2}{(s^2+\alpha^2)^2} \\ \frac{1}{2\alpha^3}(\sin\alpha t - \alpha t\cos\alpha t) \Leftrightarrow \frac{1}{(s^2+\alpha^2)^2} \end{array}\right\}$

c. $\frac{2s+3}{s^2+4.25s+1}$

d. $\frac{s^3+8s^2+24s+32}{s^2+6s+8}$

e. $e^{-2s}\frac{3}{(2s+3)^3}$

4. Use the Initial Value Theorem to find $f(0)$ given that the Laplace transform of $f(t)$ is

$$\frac{2s+3}{s^2+4.25s+1}$$

Compare your answer with that of Exercise 3(c).

5. It is known that the Laplace transform $F(s)$ has two distinct poles, one at $s = 0$, the other at $s = -1$. It also has a single zero at $s = 1$, and we know that $\lim_{t \to \infty} f(t) = 10$. Find $F(s)$ and $f(t)$.

4.6 Exercises

1. In the circuit of Figure 4.22, switch S has been closed for a long time, and opens at $t = 0$. Use the Laplace transform method to compute $i_L(t)$ for $t > 0$.

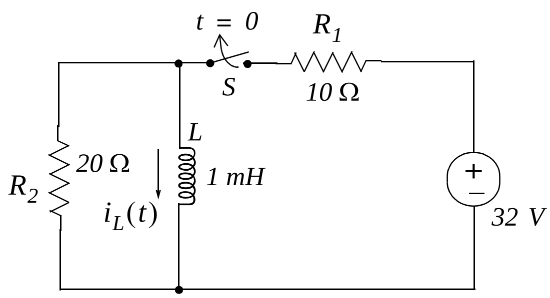

Figure 4.22. Circuit for Exercise 1

2. In the circuit of Figure 4.23, switch S has been closed for a long time, and opens at $t = 0$. Use the Laplace transform method to compute $v_c(t)$ for $t > 0$.

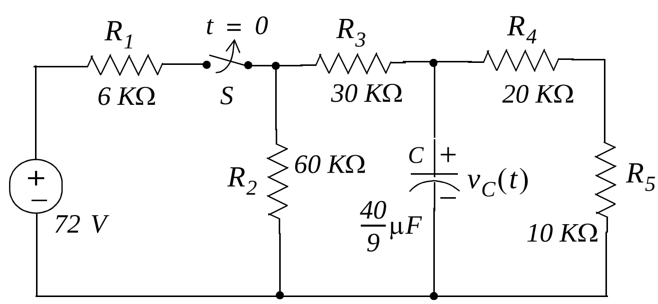

Figure 4.23. Circuit for Exercise 2

3. Use mesh analysis and the Laplace transform method, to compute $i_1(t)$ and $i_2(t)$ for the circuit of Figure 4.24, given that $i_L(0^-) = 0$ and $v_c(0^-) = 0$.

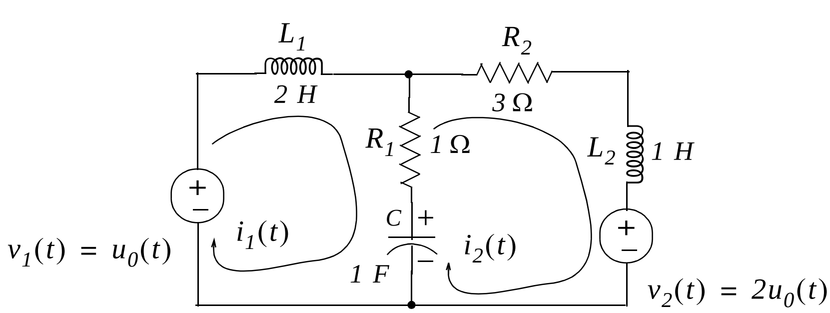

Figure 4.24. Circuit for Exercise 3

4. For the $s-domain$ circuit of Figure 4.25,

   a. compute the admittance $Y(s) = I_1(s)/V_1(s)$

   b. compute the $t-domain$ value of $i_1(t)$ when $v_1(t) = u_0(t)$, and all initial conditions are zero.

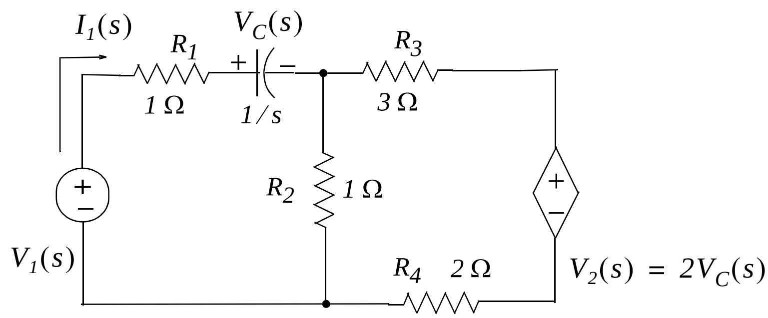

Figure 4.25. Circuit for Exercise 4

5. Derive the transfer functions for the networks (a) and (b) of Figure 4.26.

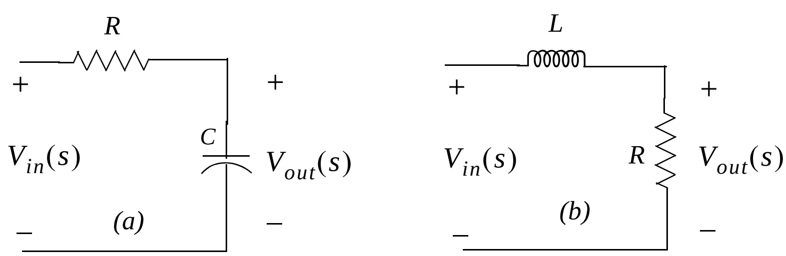

Figure 4.26. Networks for Exercise 5

6. Derive the transfer functions for the networks (a) and (b) of Figure 4.27.

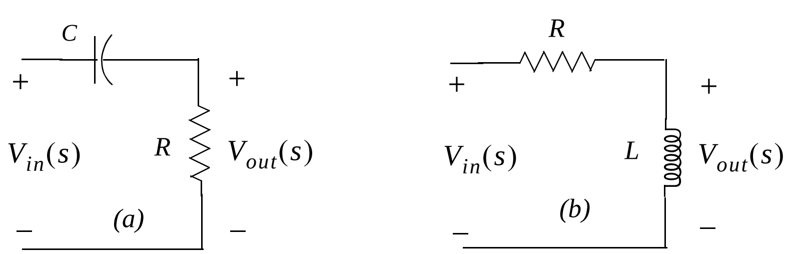

Figure 4.27. Networks for Exercise 6

7. Derive the transfer functions for the networks (a) and (b) of Figure 4.28.

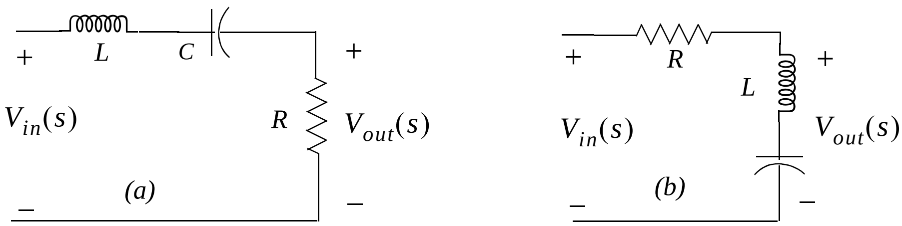

Figure 4.28. Networks for Exercise 7

8. Derive the transfer function for the networks (a) and (b) of Figure 4.29.

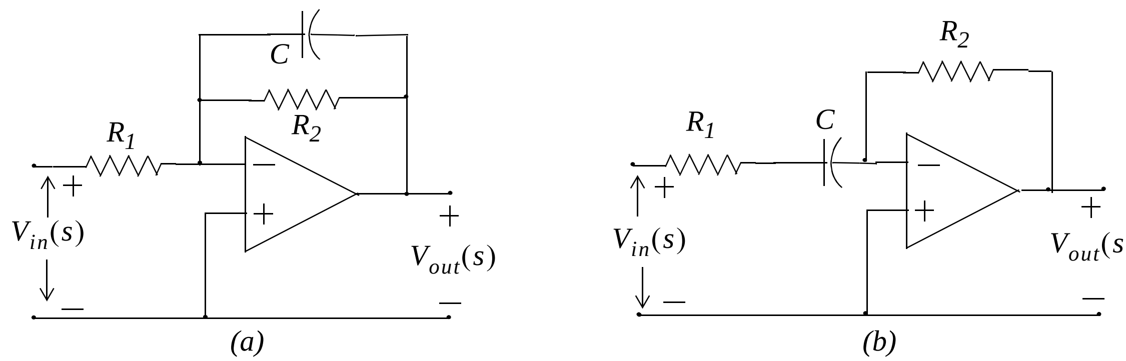

Figure 4.29. Networks for Exercise 8

9. Derive the transfer function for the network of Figure 4.30. Using MATLAB, plot $|G(s)|$ versus frequency in Hertz, on a semilog scale.

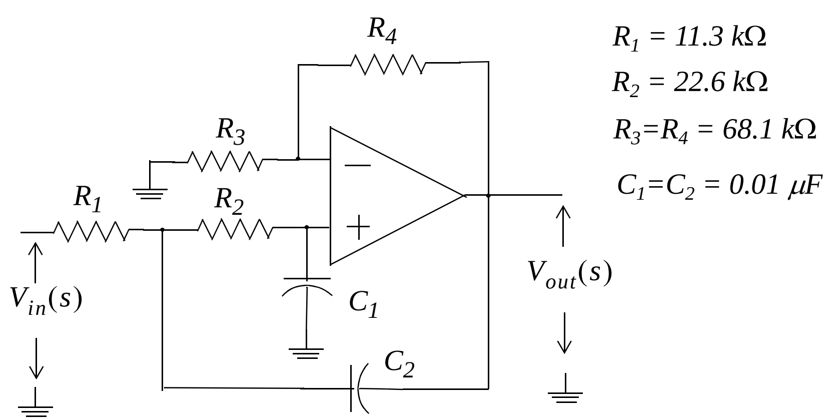

Figure 4.30. Network for Exercise 9

6.7 Exercises

1. Compute the impulse response $h(t) = i_L(t)$ in terms of R and L for the circuit of Figure 6.36. Then, compute the voltage $v_L(t)$ across the inductor.

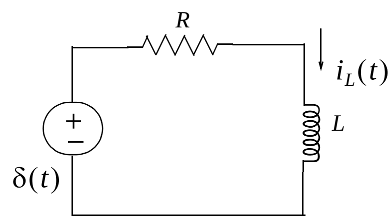

Figure 6.36. Circuit for Exercise 1

2. Repeat Example 6.4 by forming $h(t-\tau)$ instead of $u(t-\tau)$, that is, use the convolution integral

$$\int_{-\infty}^{\infty} u(\tau)h(t-\tau)d\tau$$

3. Repeat Example 6.5 by forming $h(t-\tau)$ instead of $u(t-\tau)$.

4. Compute $v_1(t)*v_2(t)$ given that

$$v_1(t) = \begin{cases} 4t & t \ge 0 \\ 0 & t < 0 \end{cases} \qquad v_2(t) = \begin{cases} e^{-2t} & t \ge 0 \\ 0 & t < 0 \end{cases}$$

5. For the series RL circuit shown in Figure 6.37, the response is the current $i_L(t)$. Use the convolution integral to find the response when the input is the unit step $u_0(t)$.

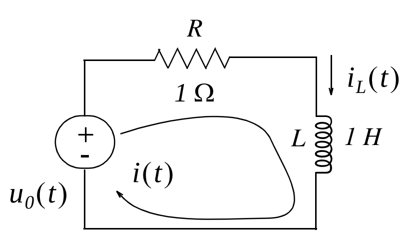

Figure 6.37. Circuit for Exercise 5

6. Compute $v_{out}(t)$ for the network of Figure 6.38 using the convolution integral, given that $v_{in}(t) = u_0(t) - u_0(t-1)$.

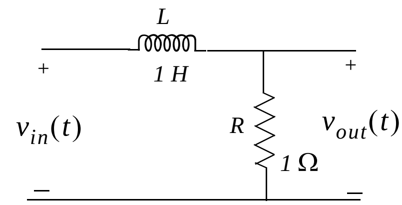

Figure 6.38. Network for Exercise 6

7. Compute $v_{out}(t)$ for the circuit of Figure 6.39 given that $v_{in}(t) = u_0(t) - u_0(t-1)$.

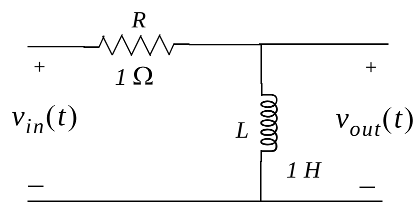

Figure 6.39. Network for Exercise 7

Hint: Use the result of Exercise 6.

7.14 Exercises

1. Compute the first 5 components of the trigonometric Fourier series for the waveform of Figure 7.47. Assume $\omega = 1$.

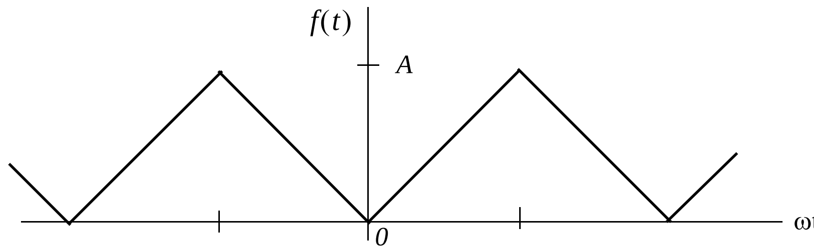

Figure 7.47. Waveform for Exercise 1

2. Compute the first 5 components of the trigonometric Fourier series for the waveform of Figure 7.48. Assume $\omega = 1$.

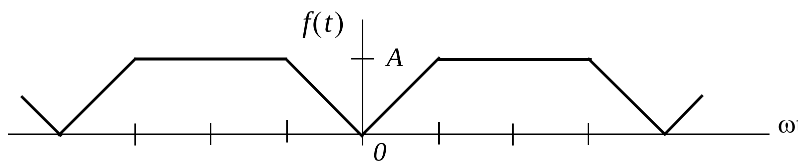

Figure 7.48. Waveform for Exercise 2

3. Compute the first 5 components of the exponential Fourier series for the waveform of Figure 7.49. Assume $\omega = 1$.

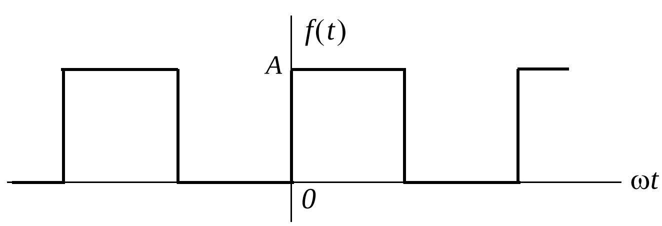

Figure 7.49. Waveform for Exercise 3

4. Compute the first 5 components of the exponential Fourier series for the waveform of Figure 7.50. Assume $\omega = 1$.

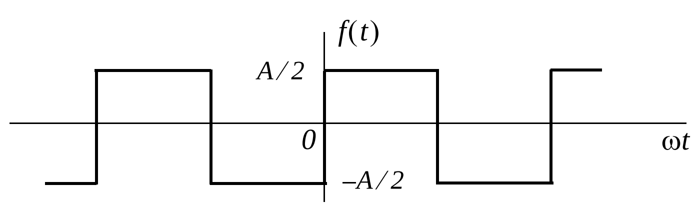

Figure 7.50. Waveform for Exercise 4

5. Compute the first 5 components of the exponential Fourier series for the waveform of Figure 7.51. Assume $\omega = 1$.

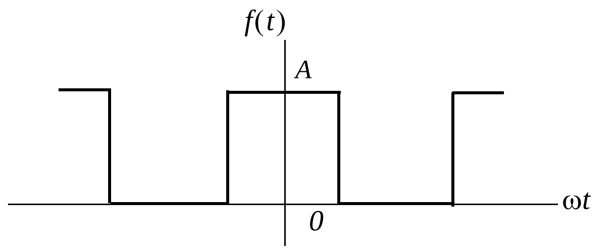

Figure 7.51. Waveform for Exercise 5

6. Compute the first 5 components of the exponential Fourier series for the waveform of Figure 7.52. Assume $\omega = 1$.

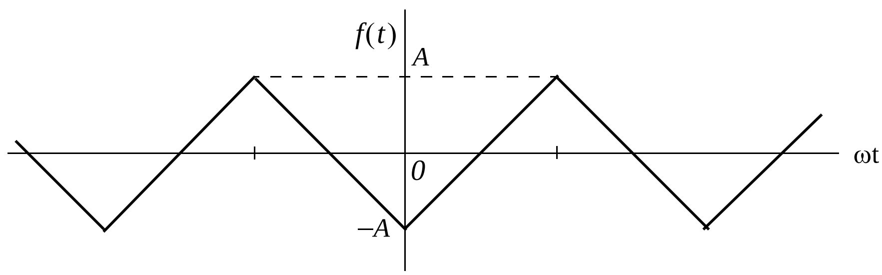

Figure 7.52. Waveform for Exercise 6

8.10 Exercises

1. Show that

$$\int_{-\infty}^{\infty} u_0(t)\delta(t)dt = 1/2$$

2. Compute

$$\mathcal{F}\{te^{-at}u_0(t)\} \quad a>0$$

3. Sketch the time and frequency waveforms of

$$f(t) = \cos\omega_0 t[u_0(t+T)-u_0(t-T)]$$

4. Derive the Fourier transform of

$$f(t) = A[u_0(t+3T)-u_0(t+T)+u_0(t-T)-u_0(t-3T)]$$

5. Derive the Fourier transform of

$$f(t) = \frac{A}{T}t[u_0(t+T)-u_0(t-T)]$$

6. Derive the Fourier transform of

$$f(t) = \left(\frac{A}{T}t+A\right)[u_0(t+T)-u_0(t)]+\left(-\frac{A}{T}t+A\right)\left[u_0(t)-u_0\left(t-\frac{T}{2}\right)\right]$$

7. For the circuit of Figure 8.21, use the Fourier transform method to compute $v_C(t)$.

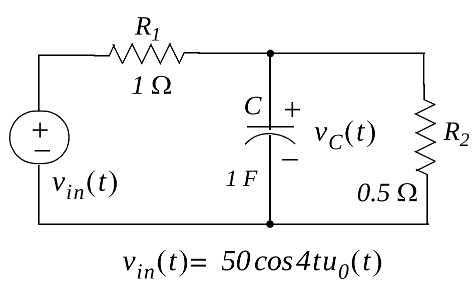

Figure 8.21. Circuit for Exercise 7

8. The input-output relationship in a certain network is

$$\frac{d^2}{dt^2}v_{out}(t)+5\frac{d}{dt}v_{out}(t)+6v_{out}(t) = 10v_{in}(t)$$

Use the Fourier transform method to compute $v_{out}(t)$ given that $v_{in}(t) = 2e^{-t}u_0(t)$.

9. In a bandpass filter, the lower and upper cutoff frequencies are $f_1 = 2\ Hz$, and $f_2 = 6\ Hz$ respectively. Compute the $1\ \Omega$ energy of the input, and the percentage that appears at the output, if the input signal is $v_{in}(t) = 3e^{-2t}u_0(t)$ volts.

10. In Example 8.4, we derived the Fourier transform pair

$$A[u_0(t+T) - u_0(t-T)] \Leftrightarrow 2AT\frac{\sin\omega T}{\omega T}$$

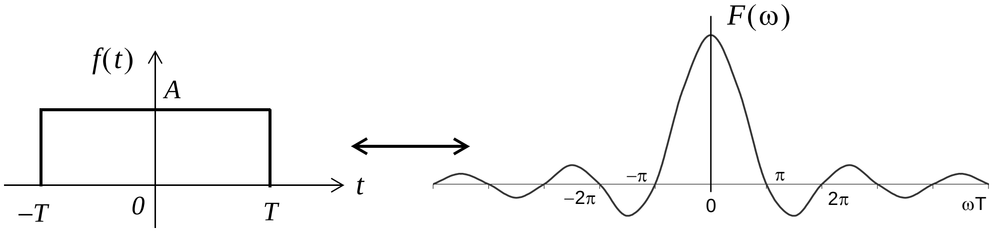

Figure 8.22. Figure for Exercise 10

Compute the percentage of the $1\ \Omega$ energy of $f(t)$ contained in the interval $-\pi/T \le \omega \le \pi/T$ of $F(\omega)$.

9.10 Exercises

1. Find the $\mathcal{Z}$ transform of the discrete time pulse $p[n]$ defined as

$$p[n] = \begin{cases} 1 & n = 0, 1, 2, \ldots, m-1 \\ 0 & otherwise \end{cases}$$

2. Find the $\mathcal{Z}$ transform of $a^n p[n]$ where $p[n]$ is defined as in Exercise 1.

3. Prove the following $\mathcal{Z}$ transform pairs:

   a. $\delta[n] \Leftrightarrow 1$

   b. $\delta[n-1] \Leftrightarrow z^{-m}$

   c. $na^n u_0[n] \Leftrightarrow \dfrac{az}{(z-a)^2}$

   d. $n^2 a^n u_0[n] \Leftrightarrow \dfrac{az(z+a)}{(z-a)^3}$

   e. $[n+1]u_0[n] \Leftrightarrow \dfrac{z^2}{(z-1)^2}$

4. Use the partial fraction expansion to find $f[n] = \mathcal{Z}^{-1}[F(z)]$ given that

$$F(z) = \frac{A}{(1-z^{-1})(1-0.5z^{-1})}$$

5. Use the partial fraction expansion method to compute the Inverse $\mathcal{Z}$ transform of

$$F(z) = \frac{z^2}{(z+1)(z-0.75)^2}$$

6. Use the Inversion Integral to compute the Inverse $\mathcal{Z}$ transform of

$$F(z) = \frac{1+2z^{-1}+z^{-3}}{(1-z^{-1})(1-0.5z^{-1})}$$

7. Use the long division method to compute the first 5 terms of the discrete time sequence whose $\mathcal{Z}$ transform is

$$F(z) = \frac{z^{-1}+z^{-2}-z^{-3}}{1+z^{-1}+z^{-2}+4z^{-3}}$$

8. a. Compute the transfer function of the difference equation

$$y[n] - y[n-1] = Tx[n-1]$$

b. Compute the response $y[n]$ when the input is $x[n] = e^{-naT}$

9. Given the difference equation

$$y[n] - y[n-1] = \frac{T}{2}\{x[n] + x[n-1]\}$$

a. Compute the discrete transfer function $H(z)$

b. Compute the response to the input $x[n] = e^{-naT}$

10. A discrete time system is described by the difference equation

$$y[n] + y[n-1] = x[n]$$

where

$$y[n] = 0 \ \ for \ \ n < 0$$

a. Compute the transfer function $H(z)$

b. Compute the impulse response $h[n]$

c. Compute the response when the input is $x[n] = 10 \ \ for \ \ n \ge 0$

11. Given the discrete transfer function

$$H(z) = \frac{z+2}{8z^2 - 2z - 3}$$

write the difference equation that relates the output $y[n]$ to the input $x[n]$.

10.8 Exercises

1. Compute the DFT of the sequence $x[0] = x[1] = 1$, $x[2] = x[3] = -1$

2. A square waveform is represented by the discrete time sequence

$$x[0] = x[1] = x[2] = x[3] = 1 \text{ and } x[4] = x[5] = x[6] = x[7] = -1$$

Use MATLAB to compute and plot the magnitude $|X[m]|$ of this sequence.

3. Prove that

a. $x[n]cos\frac{2\pi kn}{N} \Leftrightarrow \frac{1}{2}\{X[m-k] + X[m+k]\}$

b. $x[n]sin\frac{2\pi kn}{N} \Leftrightarrow \frac{1}{j2}\{X[m-k] + X[m+k]\}$

4. The signal flow graph of Figure 10.6 is a decimation in time, natural-input, shuffled-output type FFT algorithm. Using this graph and relation (10.69), compute the frequency component $X[3]$. Verify that this is the same as that found in Example 10.5.

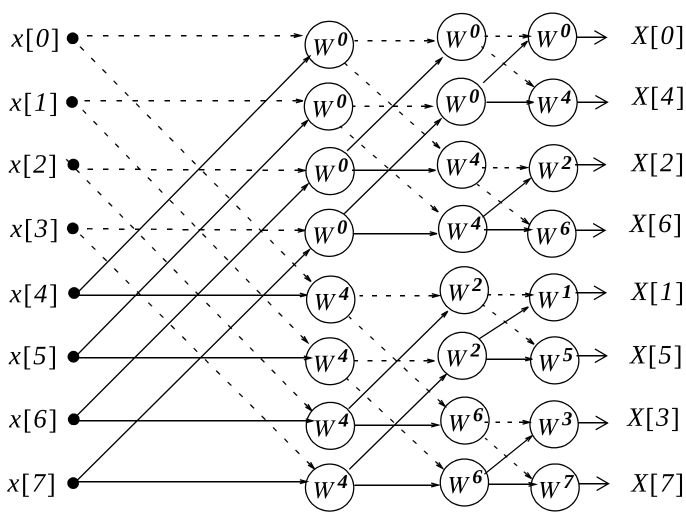

Figure 10.6. Signal flow graph for Exercise 4

5. The signal flow graph of Figure 10.7 is a decimation in frequency, natural input, shuffled output type *FFT* algorithm. There are two equations that relate successive columns. The first is

$$Y_{dash}(R, C) = Y_{dash}(R_i, C-1) + Y_{dash}(R_j, C-1)$$

and it is used with the nodes where two dashed lines terminate on them.

The second equation is

$$Y_{sol}(R, C) = W^m[Y_{sol}(R_i, C-1) - Y_{sol}(R_j, C-1)]$$

and it is used with the nodes where two solid lines terminate on them. The number inside the circles denote the power of W_N, and the minus (–) sign below serves as a reminder that the bracketed term of the second equation involves a subtraction. Using this graph and the above equations, compute the frequency component $X[3]$. Verify that this is the same as in Example 10.5.

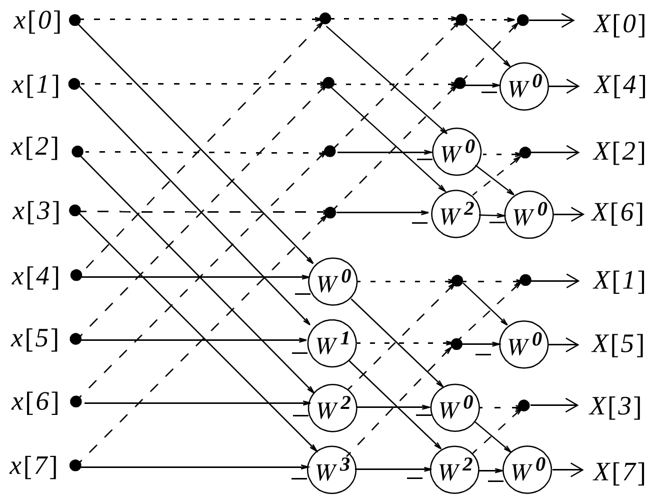

Figure 10.7. Signal flow graph for Exercise 5

11.10 Exercises

1. The circuit of Figure 11.39 is a VCVS second-order high-pass filter whose transfer function is

$$G(s) = \frac{V_{out}(s)}{V_{in}(s)} = \frac{Ks^2}{s^2 + (a/b)\omega_C s + (1/b)\omega_C^2}$$

and for given values of a, b, and desired cutoff frequency ω_C, we can calculate the values of C_1, C_2, R_1, R_2, R_3, and R_4 to achieve the desired cutoff frequency ω_C.

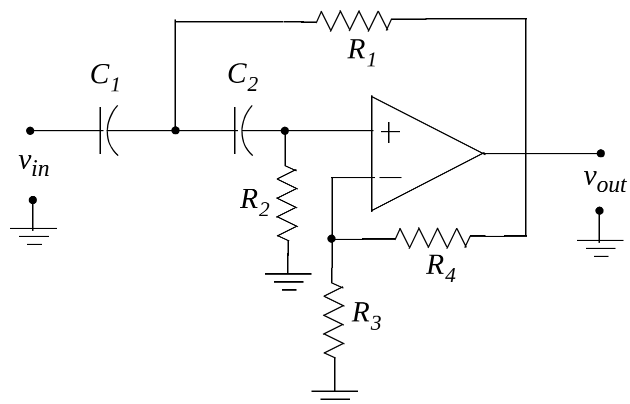

Figure 11.39. Circuit for Exercise 1

For this circuit,

$$R_2 = \frac{4b}{C_1\left\{a + [\sqrt{a^2 + 8b(K-1)}]\right\}\omega_C}$$

$$R_1 = \frac{b}{C_1^2 R_2 \omega_C^2}$$

$$R_3 = \frac{KR_2}{K-1}, \qquad K \neq 1$$

$$R_4 = KR_2$$

and the gain K is

$$K = 1 + R_4/R_3$$

Using these relations, compute the appropriate values of the resistors to achieve the cutoff frequency $f_C = 1\ KHz$. Choose the capacitors as $C_1 = 10/f_C\ \mu F$ and $C_2 = C_1$. Plot $|G(s)|$ versus frequency.

Solution using MATLAB is highly recommended.

2. The circuit of Figure 11.40 is a VCVS second-order band-pass filter whose transfer function is

$$G(s) = \frac{V_{out}(s)}{V_{in}(s)} = \frac{K[BW]s}{s^2 + [BW]s + \omega_0^2}$$

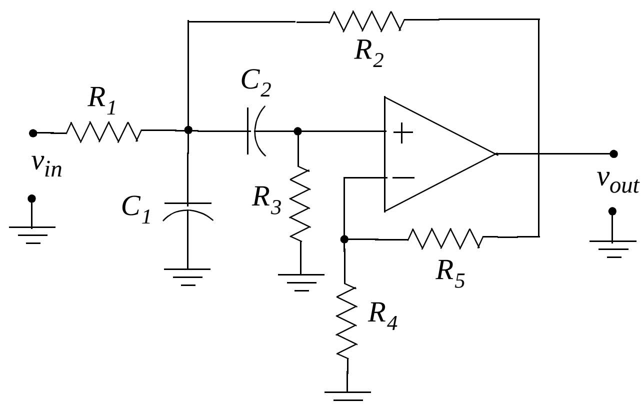

Figure 11.40. Circuit for Exercise 2

Let $\omega_0 = center\ frequency$, $\omega_2 = upper\ cutoff\ frequency$, $\omega_1 = lower\ cutoff\ frequency$, $Bandwidth\ BW = \omega_2 - \omega_1$, and $Quality\ Factor\ \ Q = \omega_0 / BW$

We can calculate the values of C_1, C_2, R_1, R_2, R_3, and R_4 to achieve the desired centered frequency ω_0 and bandwidth BW. For this circuit,

$$R_1 = \frac{2Q}{C_1 \omega_0 K}$$

$$R_2 = \frac{2Q}{C_1 \omega_0 \left\{ -1 + \sqrt{(K-1)^2 + 8Q^2} \right\}}$$

$$R_3 = \frac{1}{C_1^2 \omega_0^2} \left(\frac{1}{R_1} + \frac{1}{R_2} \right)$$

$$R_4 = R_5 = 2R_3$$

Using these relations, compute the appropriate values of the resistors to achieve center frequency $f_0 = 1\ KHz$, $Gain\ K = 10$, and $Q = 10$.

Choose the capacitors as $C_1 = C_2 = 0.1\ \mu F$. Plot $|G(s)|$ versus frequency.

Solution using MATLAB is highly recommended.

3. The circuit of Figure 11.41 is a $VCVS$ second-order band elimination filter whose transfer function is

$$G(s) = \frac{V_{out}(s)}{V_{in}(s)} = \frac{K(s^2 + \omega_0^2)}{s^2 + [BW]s + \omega_0^2}$$

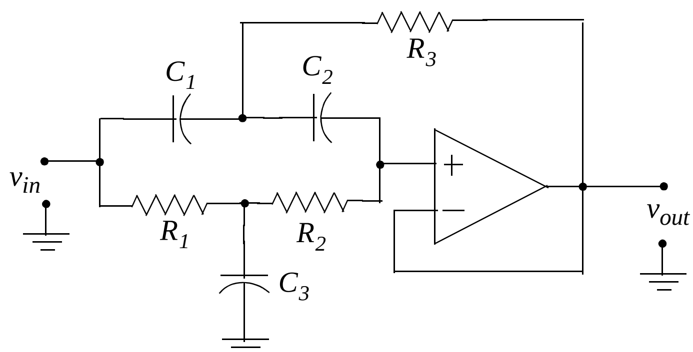

Figure 11.41. Circuit for Exercise 3

Let $\omega_0 = center\ frequency$, $\omega_2 = upper\ cutoff\ frequency$, $\omega_1 = lower\ cutoff\ frequency$, $Bandwidth\ BW = \omega_2 - \omega_1$, $Quality\ Factor\ Q = \omega_0 / BW$, and gain $K = 1$

We can calculate the values of $C_1, C_2, R_1, R_2, R_3,$ and R_4 to achieve the desired centered frequency ω_0 and bandwidth BW. For this circuit,

$$R_1 = \frac{1}{2\omega_0 QC_1}$$

$$R_2 = \frac{2Q}{\omega_0 C_1}$$

$$R_3 = \frac{2Q}{C_1\omega_0(4Q^2 + 1)}$$

The gain K must be unity, but Q can be up to 10. Using these relations, compute the appropriate values of the resistors to achieve center frequency $f_0 = 1\ KHz$, $Gain\ K = 1$ and $Q = 10$.

Choose the capacitors as $C_1 = C_2 = 0.1\ \mu F$ and $C_3 = 2C_1$. Plot $|G(s)|$ versus frequency.

Solution using MATLAB is highly recommended.

4. The circuit of Figure 11.42 is a MFB second-order *all-pass filter* whose transfer function is

$$G(s) = \frac{V_{out}(s)}{V_{in}(s)} = \frac{K(s^2 - a\omega_0 s + b\omega_0^2)}{s^2 + a\omega_0 s + b\omega_0^2}$$

where the gain $K = constant$, $(0 < K < 1)$, and the phase is given by

$$\phi(\omega) = -2tan^{-1}\left(\frac{a\omega_0\omega}{b\omega_0^2 - \omega^2}\right)$$

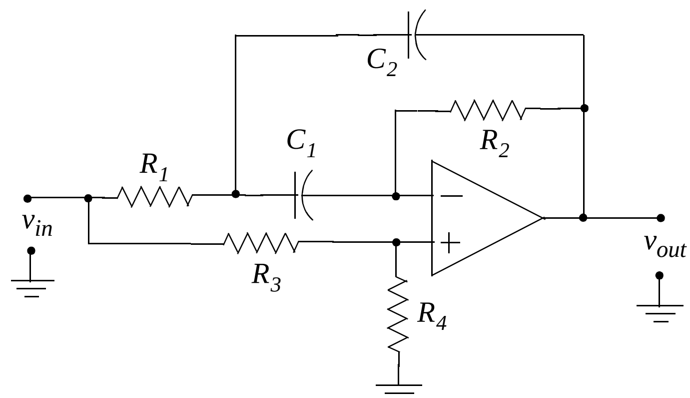

Figure 11.42. Circuit for Exercise 4

The coefficients a and b can be found from

$$\phi_0 = \phi(\omega_0) = -2tan^{-1}\left(\frac{a}{b-1}\right)$$

For arbitrary values of $C_1 = C_2$, we can compute the resistances from

$$R_2 = \frac{2}{a\omega_0 C_1}$$

$$R_1 = \frac{(1-K)R_2}{4K}$$

$$R_3 = \frac{R_2}{K}$$

$$R_4 = \frac{R_2}{1-K}$$

For $0 < \phi_0 < 180°$, we compute the coefficient a from

$$a = \frac{1-K}{2Ktan(\phi_0/2)}\left[-1 + \sqrt{1 + \frac{4K}{1-K} \cdot tan^2(\phi_0/2)}\right]$$

and for $-180° < \phi_0 < 0°$, from

$$a = \frac{1-K}{2K\tan(\phi_0/2)}\left[-1-\sqrt{1+\frac{4K}{1-K}\cdot\tan^2(\phi_0/2)}\right]$$

Using these relations, compute the appropriate values of the resistors to achieve a phase shift $\phi_0 = -90°$ at $f_0 = 1\ KHz$ with $K = 0.75$.

Choose the capacitors as $C_1 = C_2 = 0.01\ \mu F$ and plot phase versus frequency.

Solution using MATLAB is highly recommended.

5. The *Bessel filter* of Figure 11.43 has the same configuration as the low-pass filter of Example 11.3, and achieves a relatively constant time delay over a range $0 < \omega < \omega_0$. The second-order transfer function of this filter is

$$G(s) = \frac{V_{out}(s)}{V_{in}(s)} = \frac{3K\omega_0^2}{s^2+3\omega_0 s+3\omega_0^2}$$

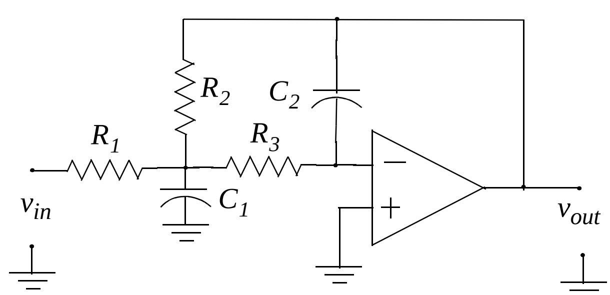

Figure 11.43. Circuit for Exercise 5

where K is the gain and the time delay T_0 at $\omega_0 = 2\pi f_0$ is given as

$$T_0 = T(\omega_0) = \frac{12}{13\omega_0}\ seconds$$

We recognize the transfer function $|G(s)|$ above as that of a low-pass filter where $a = b = 3$ and the substitution of $\omega_0 = \omega_C$. Therefore, we can use a low-pass filter circuit such as that of Figure 11.43, to achieve a constant delay T_0 by specifying the resistor and capacitor values of the circuit.

The resistor values are computed from

$$R_2 = \frac{2(K+1)}{(aC_1 + \sqrt{a^2C_1^2 - 4bC_1C_2(K-1)})\omega_0}$$

$$R_1 = \frac{R_2}{K}$$

$$R_3 = \frac{1}{bC_1C_2R_2\omega_0^2}$$

Using these relations, compute the appropriate values of the resistors to achieve a time delay $T_0 = 100\ \mu s$ with $K = 2$. Use capacitors $C_1 = 0.01\ \mu F$ and $C_2 = 0.002\ \mu F$. Plot $|G(s)|$ versus frequency.

Solution using MATLAB is highly recommended.

6. Derive the transfer function of a fourth-order Butterworth filter with $\omega_C = 1\ rad/s$.

7. Derive the amplitude-squared function for a third-order Type I Chebyshev low-pass filter with $1.5\ dB$ pass band ripple and cutoff frequency $\omega_C = 1\ rad/s$.

8. Use MATLAB to derive the transfer function $G(z)$ and plot $|G(z)|$ versus ω for a two-pole, Type I Chebyshev high-pass digital filter with sampling period $T_S = 0.25\ s$. The equivalent analog filter cutoff frequency is $\omega_C = 4\ rad/s$ and has $3\ dB$ pass band ripple. Compute the coefficients of the numerator and denominator and plot $|G(z)|$ with and without pre-warping.