Search Trees

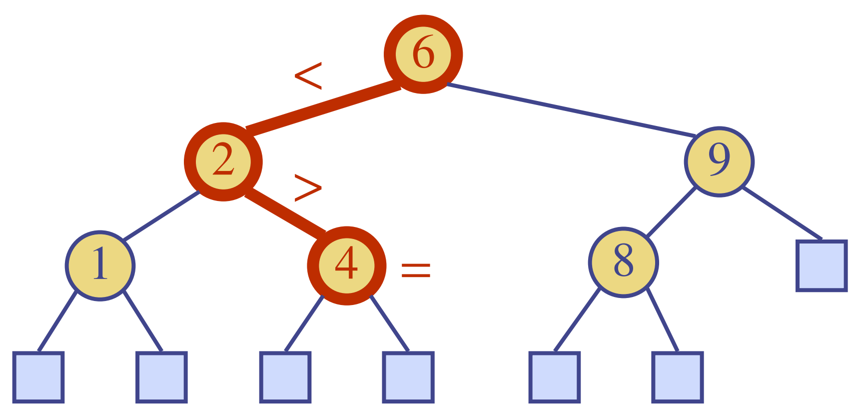
Data structures and Algorithms

Acknowledgement:
These slides are adapted from slides provided with *Data Structures and Algorithms in C++*
Goodrich, Tamassia and Mount (Wiley, 2004)

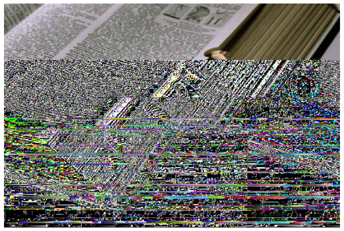
Outline

- Binary Search Trees
- AVL Trees
- (2,4) Trees
- Red-Black Trees

Binary Search Trees



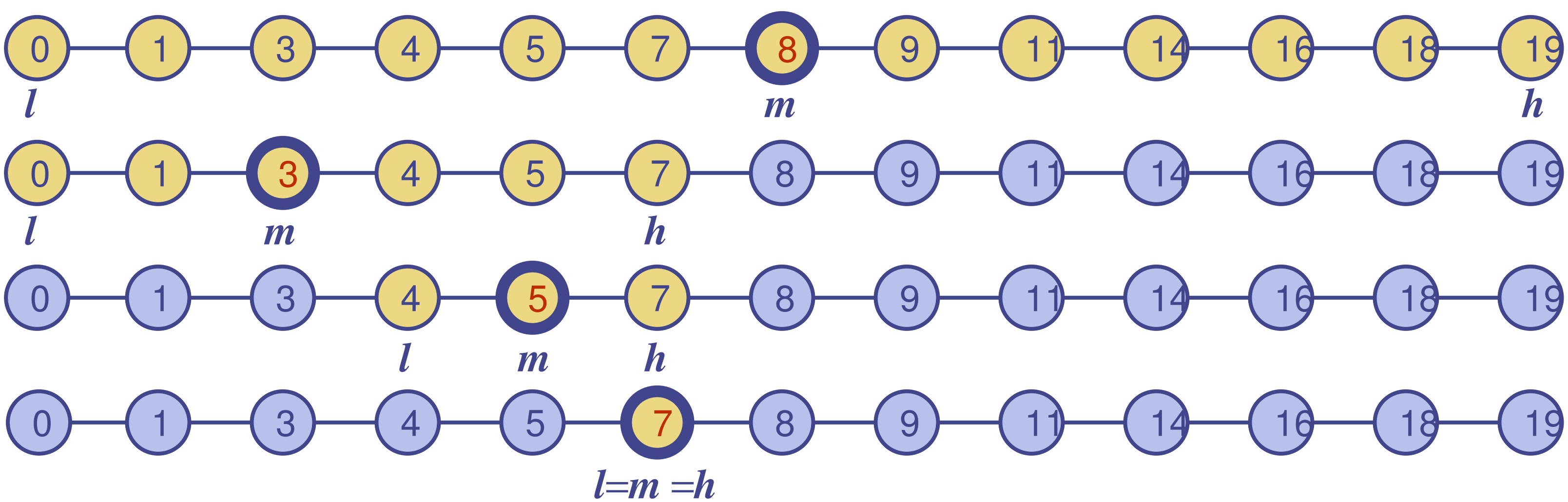
Ordered Dictionaries



- Keys are assumed to come from a total order.
- New operations:
  - first(): first entry in the dictionary ordering
  - last(): last entry in the dictionary ordering
  - successors(k): iterator of entries with keys greater than or equal to k; increasing order
  - predecessors(k): iterator of entries with keys less than or equal to k; decreasing order

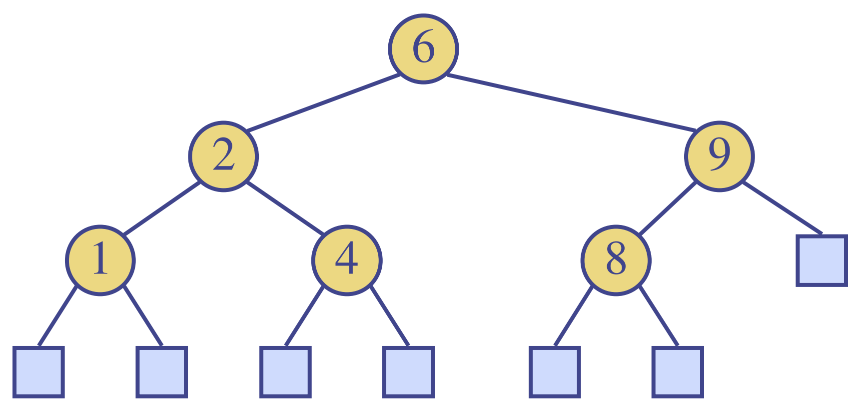
Binary Search

- Binary search can perform operation find(k) on a dictionary implemented by means of an array-based sequence, sorted by key
  - similar to the high-low game
  - at each step, the number of candidate items is halved
  - terminates after O(log n) steps
- Example: find(7)



Binary Search Trees

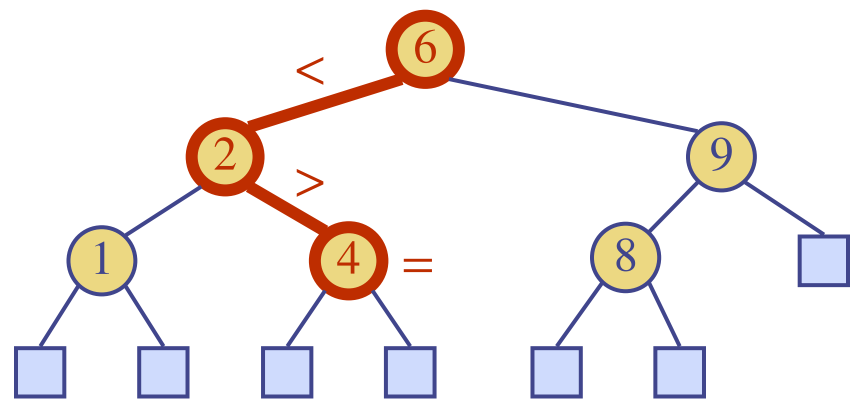
- A binary search tree is a binary tree storing keys (or key-value entries) at its internal nodes and satisfying the following property:
  - Let u, v, and w be three nodes such that u is in the left subtree of v and w is in the right subtree of v. We have
    $key(u) \leq key(v) \leq key(w)$
- External nodes do not store items
- An inorder traversal of a binary search trees visits the keys in increasing order



Search

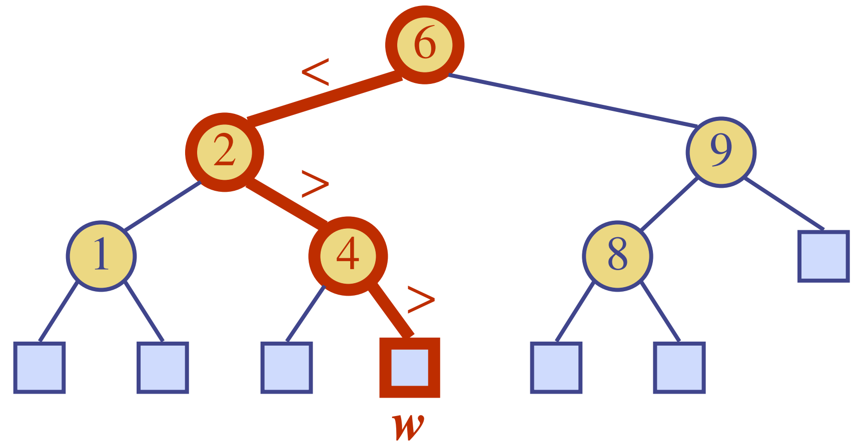
- To search for a key k, we trace a downward path starting at the root
- The next node visited depends on the outcome of the comparison of k with the key of the current node
- If we reach a leaf, the key is not found and we return null
- Example: find(4):
  - Call TreeSearch(4,root)

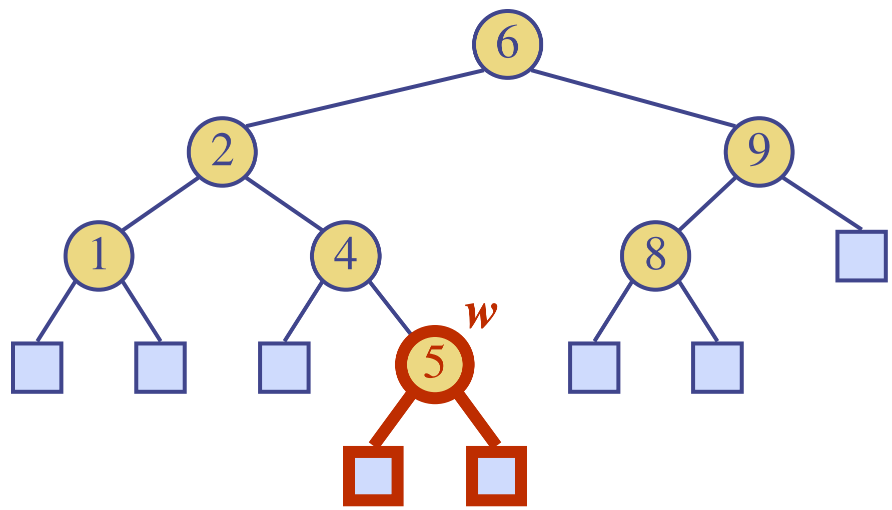
```
Algorithm TreeSearch(k, v)
    if T.isExternal (v)
        return null
    if k < key(v)
        return TreeSearch(k, T.left(v))
    else if k = key(v)
        return v
    else { k > key(v) }
        return TreeSearch(k, T.right(v))
```



Insertion

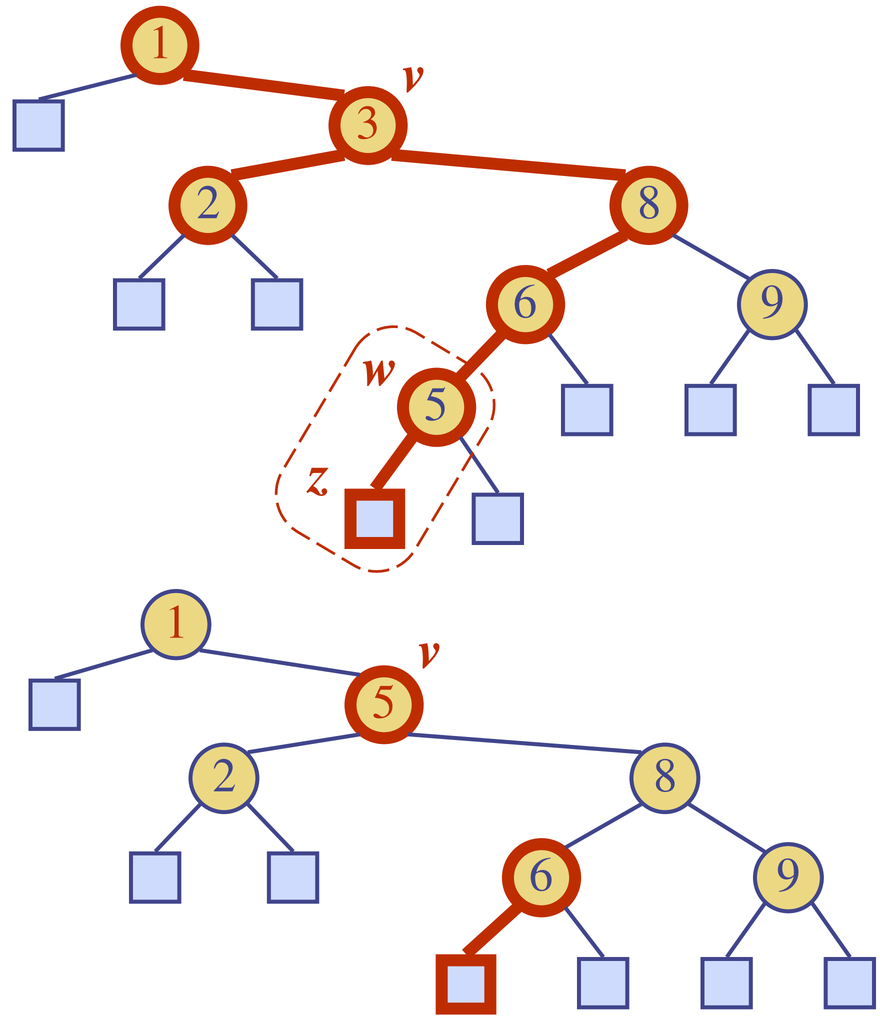
- To perform operation insert(k, o), we search for key k (using TreeSearch)
- Assume k is not already in the tree, and let let w be the leaf reached by the search
- We insert k at node w and expand w into an internal node
- Example: insert 5





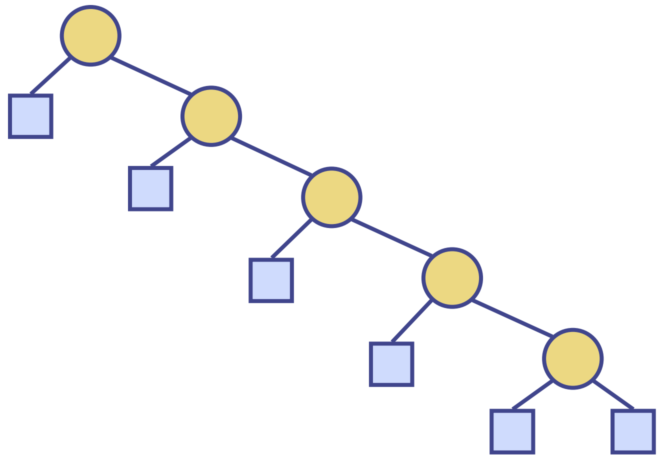
Deletion (cont.)

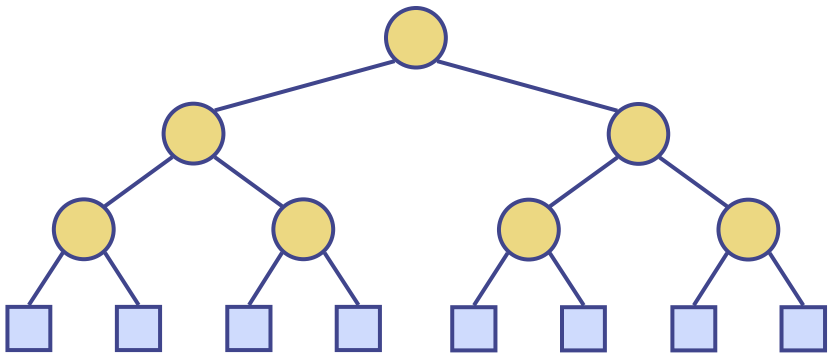
- We consider the case where the key k to be removed is stored at a node v whose children are both internal
  - we find the internal node w that follows v in an inorder traversal
  - we copy $key(w)$ into node v
  - we remove node w and its left child z (which must be a leaf) by means of operation removeExternal(z)
- Example: remove 3



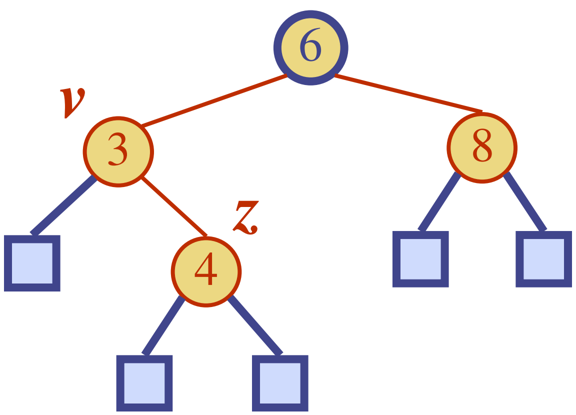
Performance

- Consider a dictionary with n items implemented by means of a binary search tree of height h
  - the space used is $O(n)$
  - methods find, insert and remove take $O(h)$ time
- The height h is $O(n)$ in the worst case and $O(\log n)$ in the best case



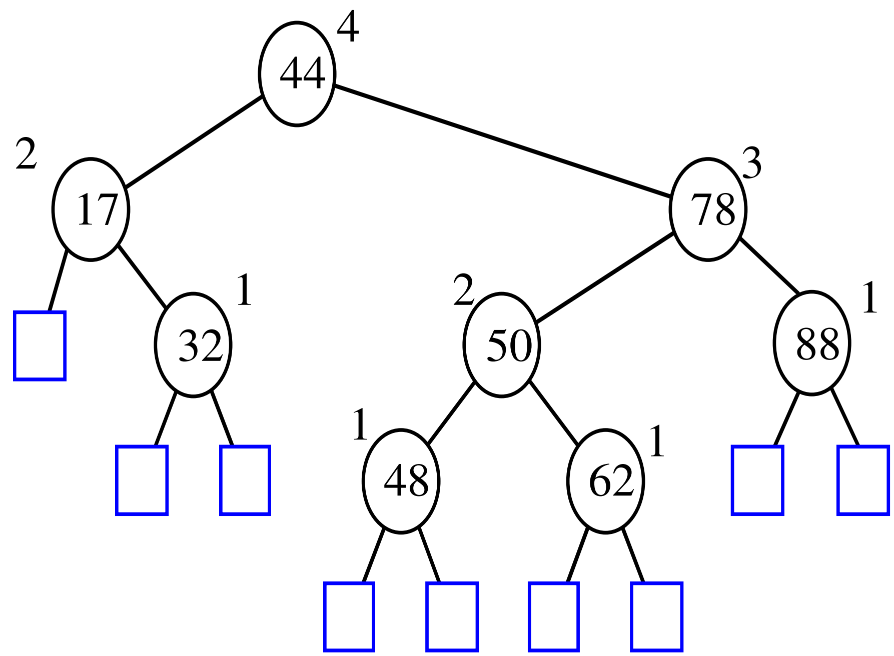


AVL Trees



AVL Tree Definition

- **AVL trees are balanced.**
- An AVL Tree is a ***binary search tree*** such that for every internal node v of T, the *heights of the children of v can differ by at most 1.*

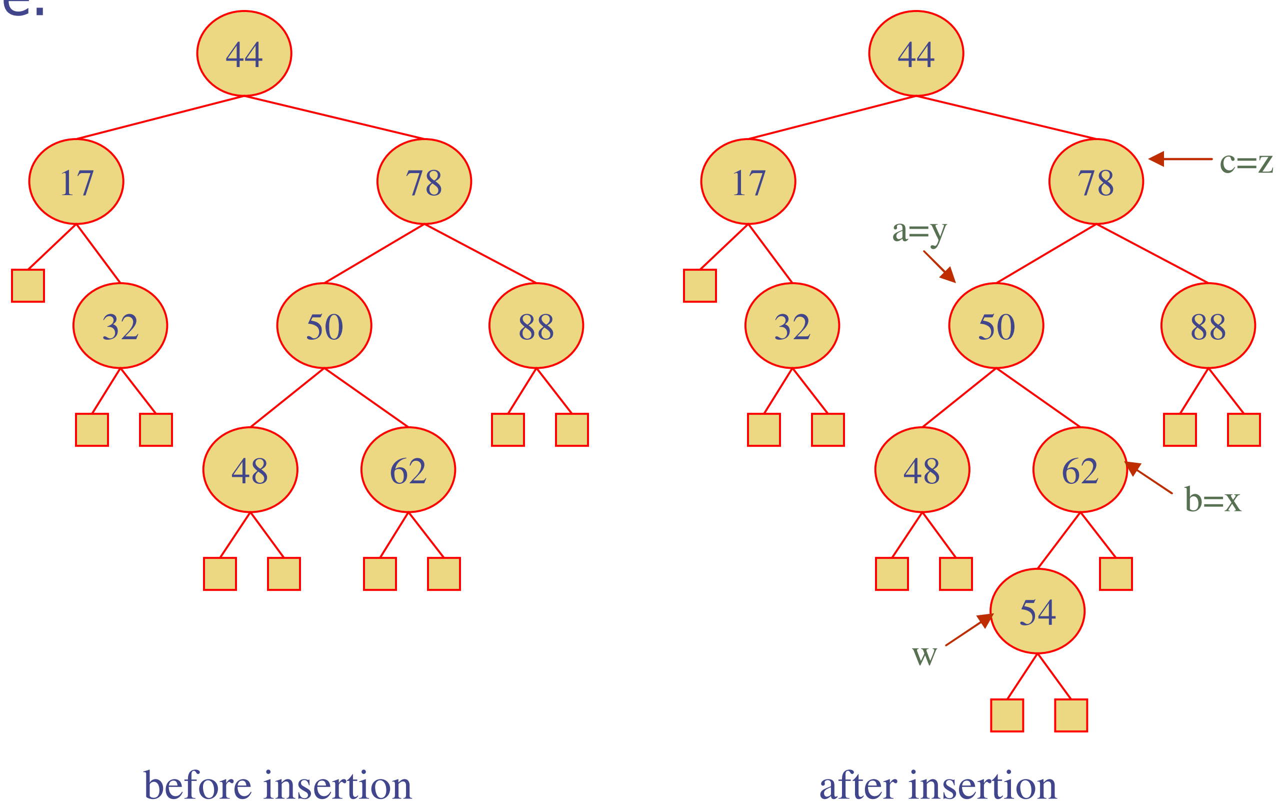


An example of an AVL tree where the heights are shown next to the nodes:

12

Insertion in an AVL Tree

- Insertion is as in a binary search tree
- Always done by expanding an external node.
- Example:



4 cases of unbalanced trees

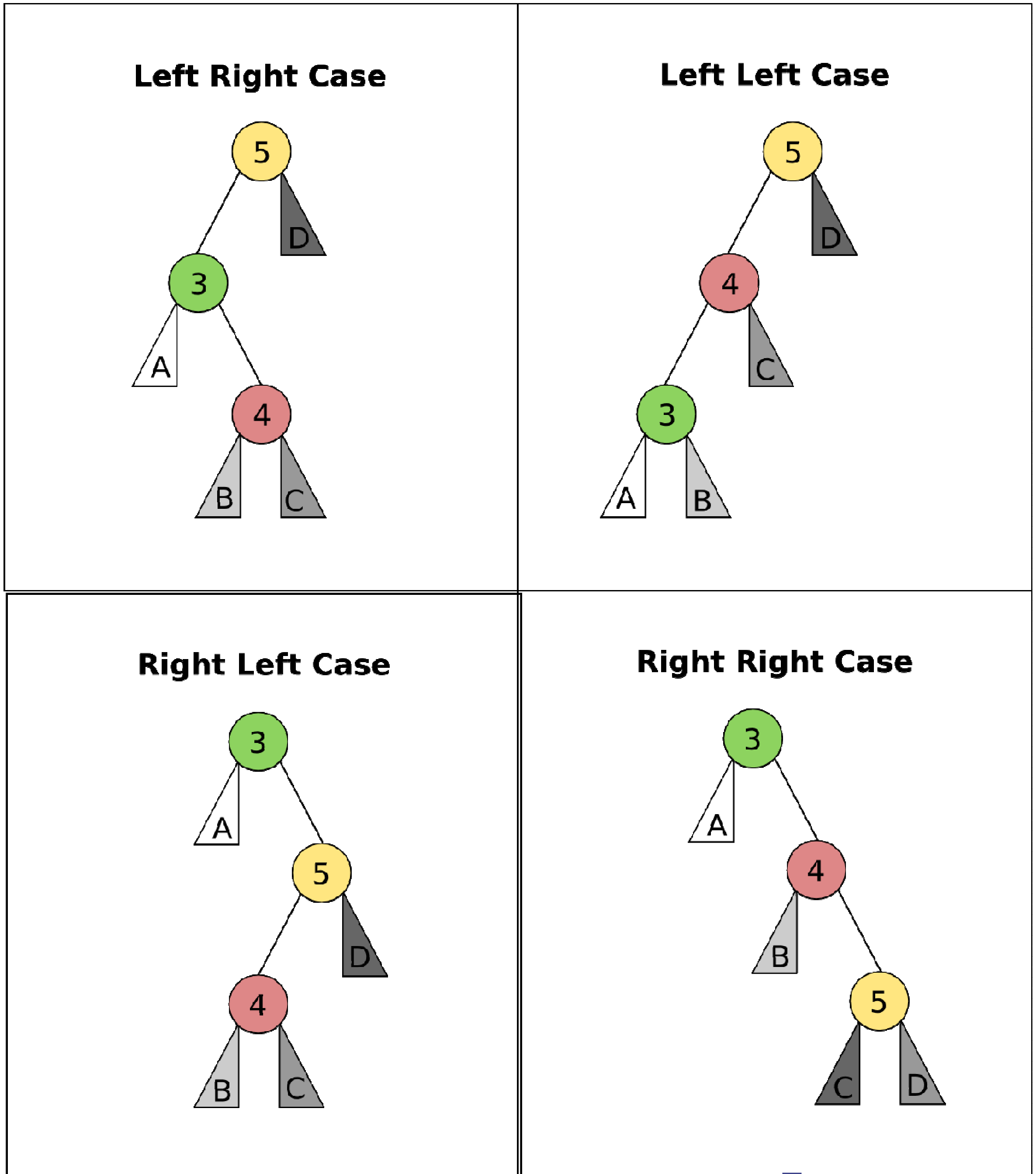
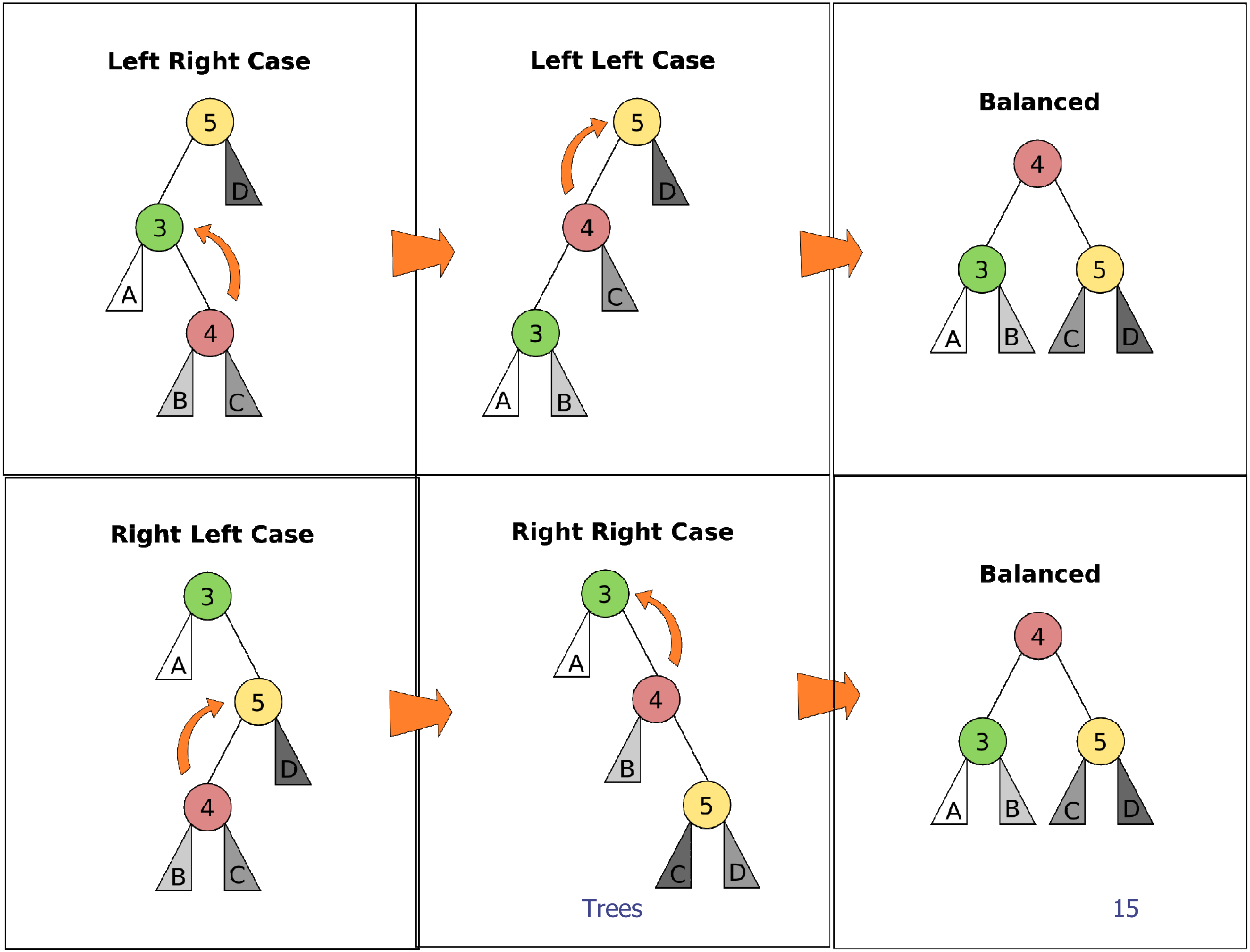
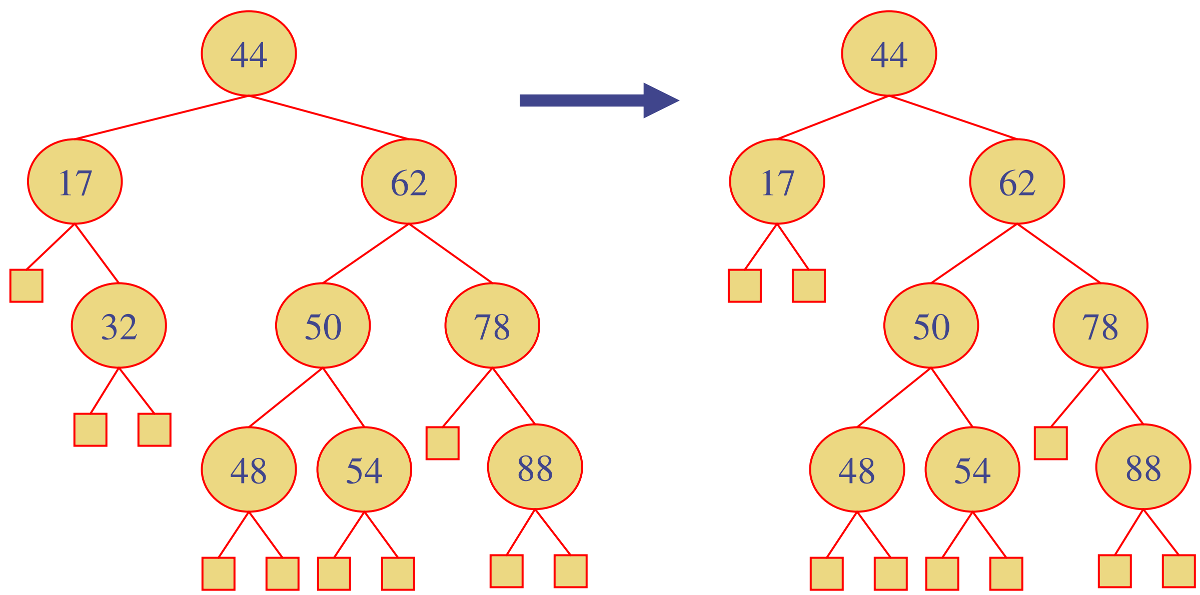


Diagram adapted from Wikipedia's original


Left Right Case
5
3
4
A
B
C
D
Left Left Case
5
4
3
A
B
C
D
Balanced
4
3
5
A
B
C
D
Right Left Case
3
5
4
A
B
C
D
Right Right Case
3
4
5
A
B
C
D
Balanced
4
3
5
A
B
C
D

Removal in an AVL Tree

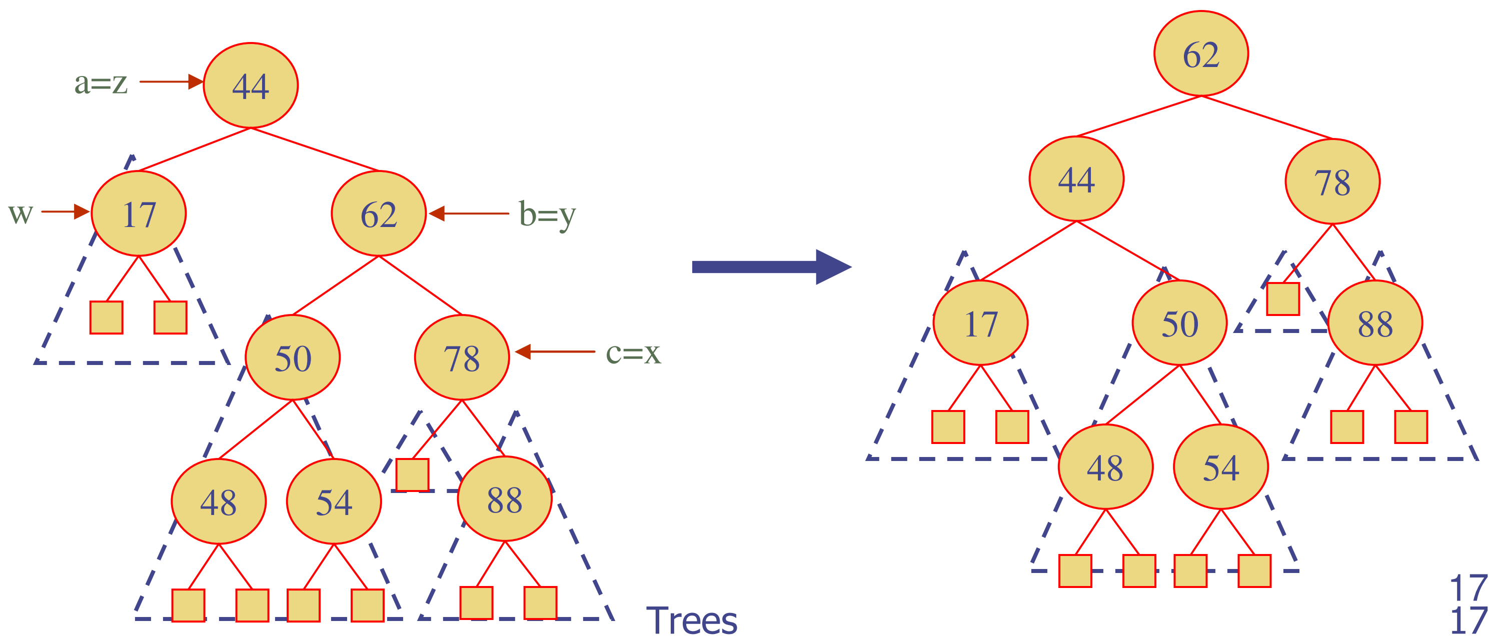
- Removal begins as in a binary search tree, which means the node removed will become an empty external node. Its parent, w, may cause an imbalance.
- Example:



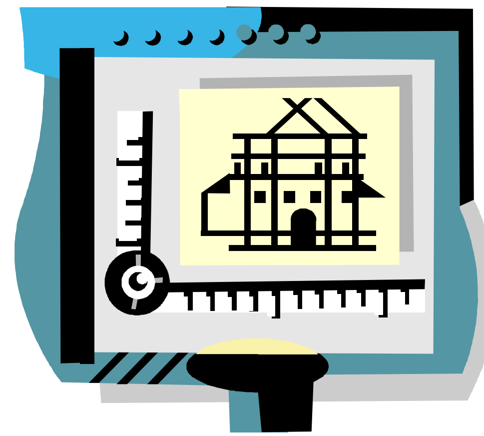
before deletion of 32 after deletion

Rebalancing after a Removal

- Let z be the first unbalanced node encountered while travelling up the tree from w. Also, let y be the child of z with the larger height, and let x be the child of y with the larger height.
- We perform restructure(x) to restore balance at z.
- As this restructuring may upset the balance of another node higher in the tree, we must continue checking for balance until the root of T is reached

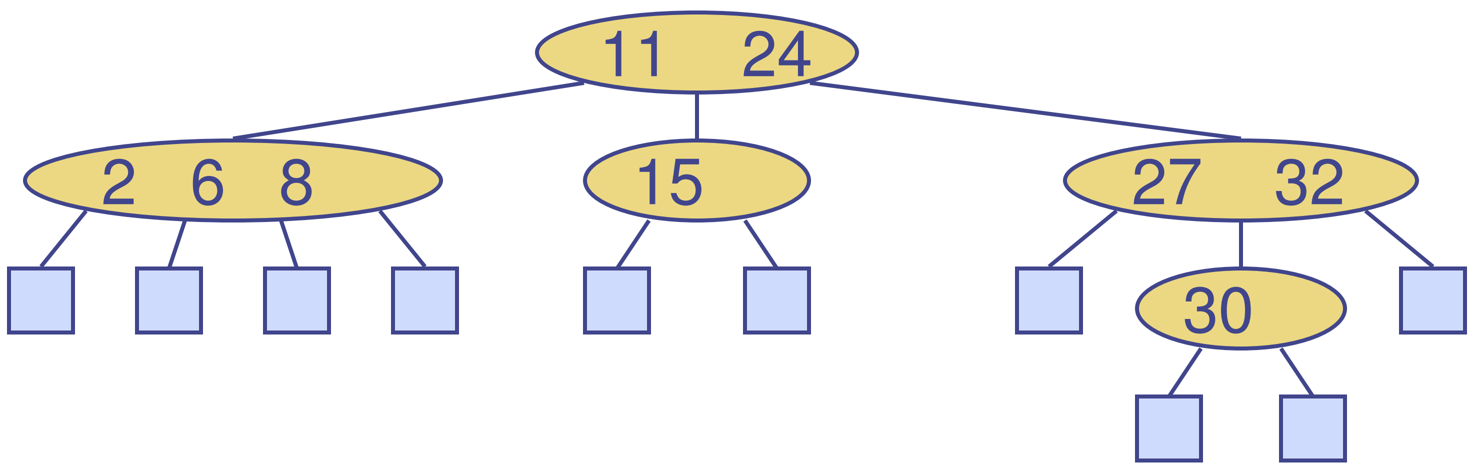


Running Times for AVL Trees



- a single restructure is O(1)
  - using a linked-structure binary tree
- find is O(log n)
  - height of tree is O(log n), no restructures needed
- insert is O(log n)
  - initial find is O(log n)
  - Restructuring up the tree, maintaining heights is O(log n)
- remove is O(log n)
  - initial find is O(log n)
  - Restructuring up the tree, maintaining heights is O(log n)

(2,4) Trees



Multi-Way Search Tree

- A multi-way search tree is an ordered tree such that
  - Each internal node has at least two children and stores $d-1$ key-element items (k_i, o_i), where d is the number of children
  - For a node with children $v_1\, v_2 \ldots v_d$ storing keys $k_1\, k_2 \ldots k_{d-1}$
    - keys in the subtree of v_1 are less than k_1
    - keys in the subtree of v_i are between k_{i-1} and k_i $(i = 2, \ldots, d-1)$
    - keys in the subtree of v_d are greater than k_{d-1}
  - The leaves store no items and serve as placeholders