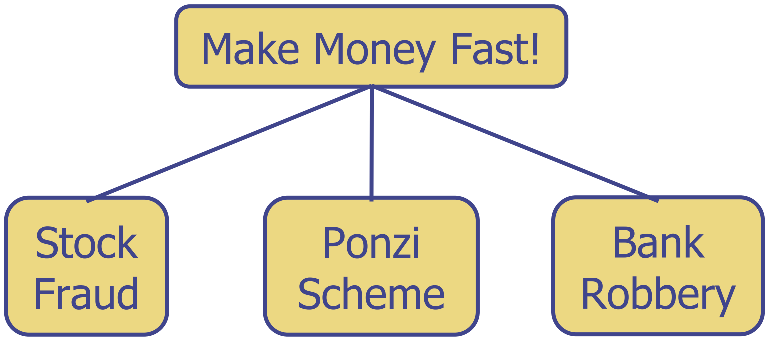


Trees

Data structures and Algorithms

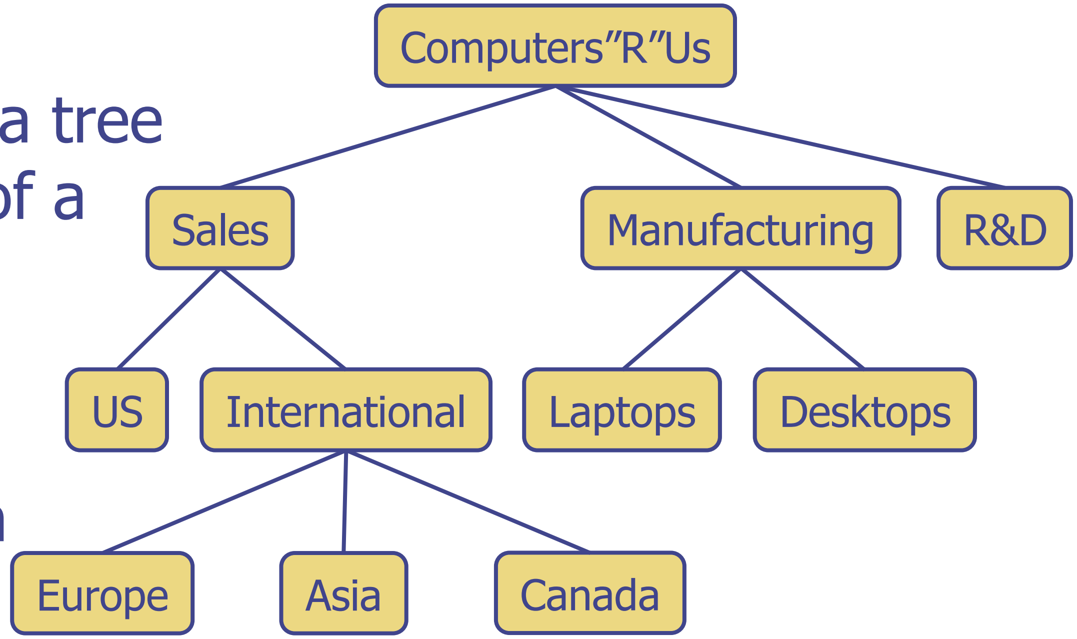
Acknowledgement:
These slides are adapted from slides provided with *Data Structures and Algorithms in C++*
Goodrich, Tamassia and Mount (Wiley, 2004)

Outline and Reading

- Tree ADT (§7.1.2)
- Preorder and postorder traversals (§7.2)
- BinaryTree ADT (§7.3)
- Inorder traversal (§7.3.6)

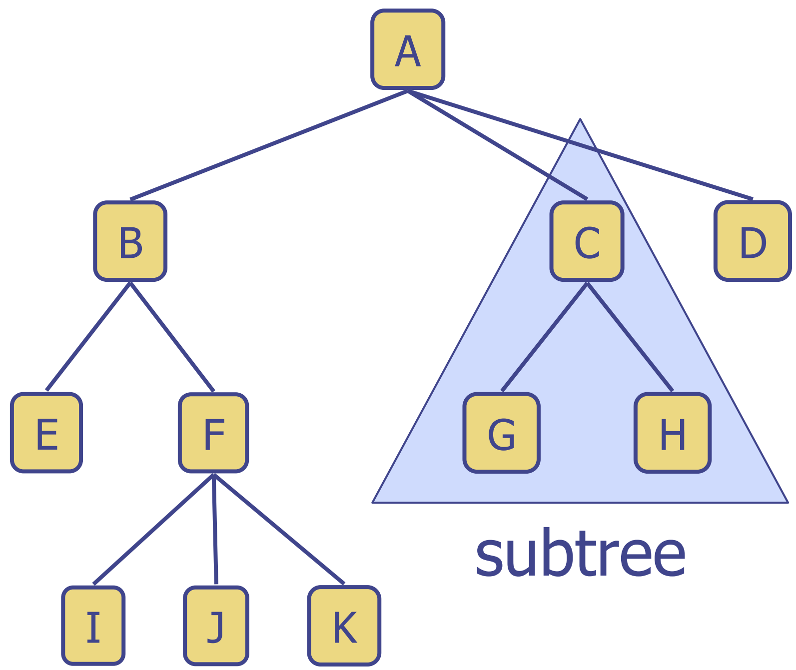
What is a Tree

- In computer science, a tree is an abstract model of a hierarchical structure
- A tree consists of nodes with a parent-child relation
- Applications:
  - Organization charts
  - File systems



Tree Terminology

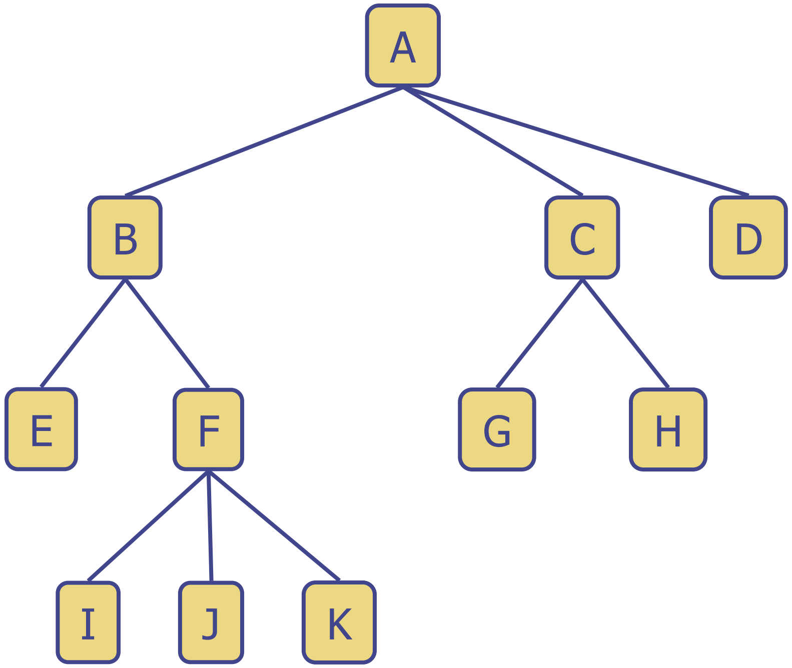
- **Root**: node without parent (A)
- **Internal node**: node with at least one child (A, B, C, F)
- **Leaf** (aka **External node**): node without children (E, I, J, K, G, H, D)
- **Ancestors** of a node: parent, grandparent, great-grandparent, etc.
- **Depth** of a node: number of ancestors
- **Height** of a tree: maximum depth of any node (3)
- **Descendant** of a node: child, grandchild, great-grandchild, etc.
- **Subtree**: tree consisting of a node and its descendants



Exercise: Trees

Answer the following questions about the tree shown on the right:

- What is the size of the tree (number of nodes)?
- Classify each node of the tree as a root, leaf, or internal node
- List the ancestors of nodes B, F, G, and A. Which are the parents?
- List the descendents of nodes B, F, G, and A. Which are the children?
- List the depths of nodes B, F, G, and A.
- What is the height of the tree?
- Draw the subtrees that are rooted at node F and at node K.



Tree ADT

We use positions to abstract nodes

Generic methods:
- integer size()
- boolean isEmpty()
- objectIterator elements()
- positionIterator positions()

Accessor methods:
- position root()
- position parent(p)
- positionIterator children(p)

Query methods:
- boolean isInternal(p)
- boolean isLeaf (p)
- boolean isRoot(p)

Update methods:
- swapElements(p, q)
- object replaceElement(p, o)

Additional update methods may be defined by data structures implementing the Tree ADT

Depth and Height

v : a node of a tree *T*.

- The ***depth*** of *v* is the number of ancestors of *v*, excluding *v* itself.
- The ***height*** of a node *v* in a tree *T* is defined recursively:
  - If *v* is an external node, then the height of *v* is 0
  - Otherwise, the height of *v* is one plus the maximum height of a child of *v*.

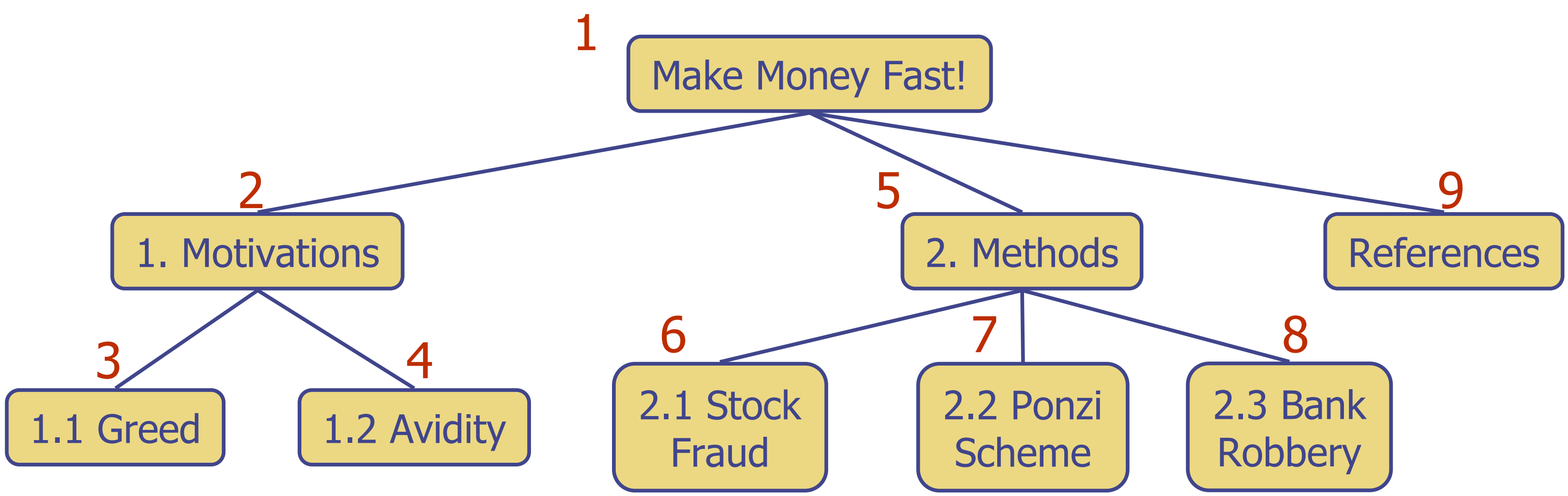
```
Algorithm depth(T, v)
   if T.isRoot (v)
      return 0
   else
      return 1 + depth (T, T.parent(v))
```

```
Algorithm height(T, v)
   if T.isExternal (v)
      return 0
   else
      h ← 0
      for each child w of v in T
         h ← max (h, height(T, w))
      return 1 + h
```

Preorder Traversal

- A *traversal* visits the nodes of a tree in a systematic manner
- In a *preorder traversal*, a node is visited before its descendants
- Application: print a structured document

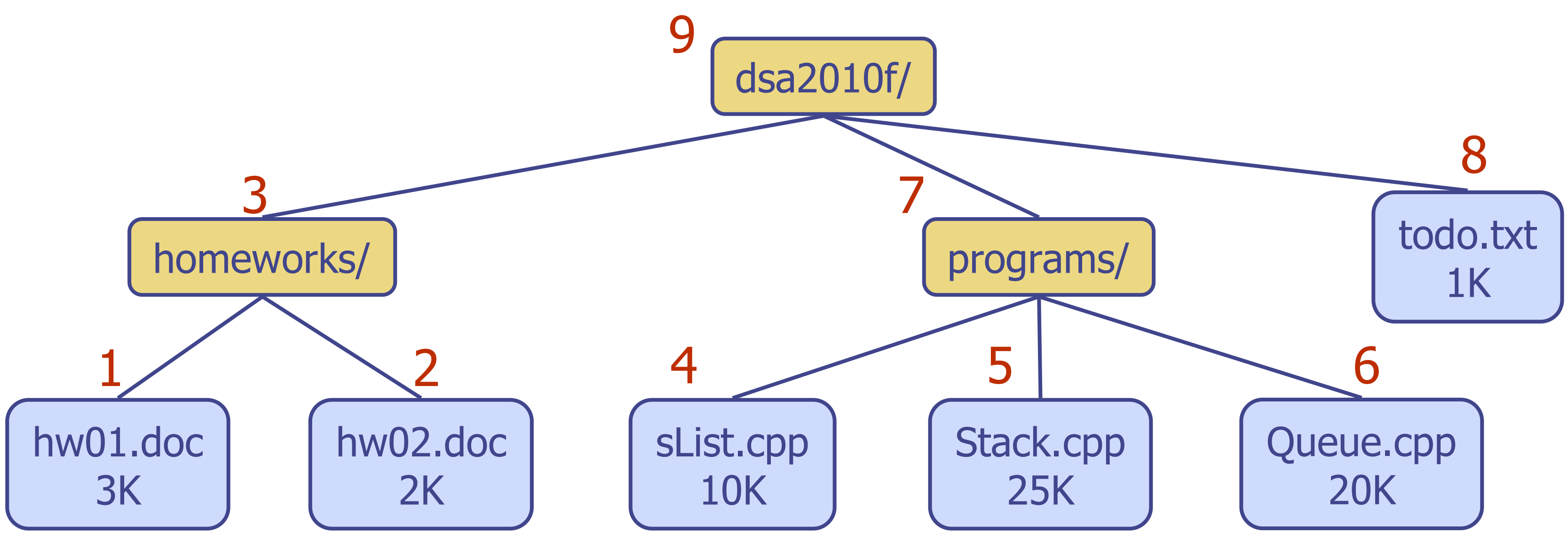
```
Algorithm preOrder(v)
    visit(v)
    for each child w of v
        preOrder (w)
```



Postorder Traversal

- In a *postorder traversal*, a node is visited after its descendants
- Application: compute space used by files in a directory and its subdirectories

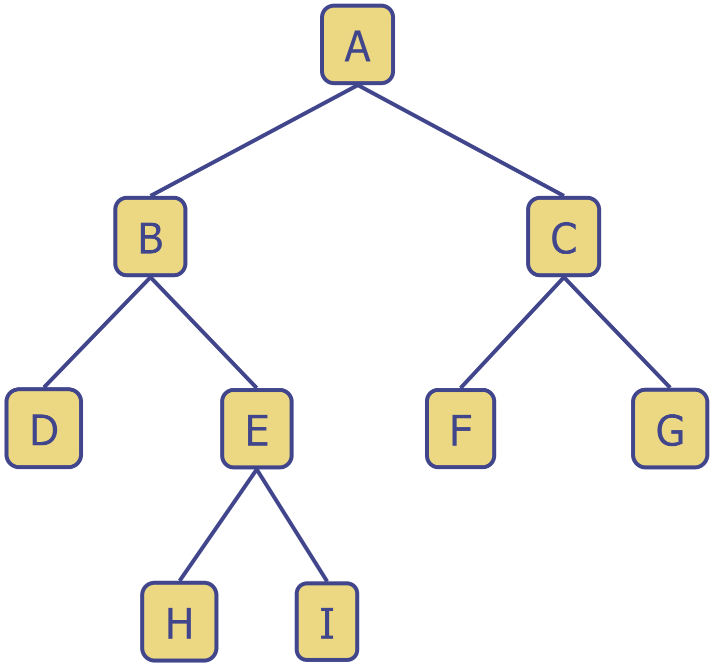
Algorithm *postOrder*(*v*)
  for each child *w* **of** *v*
    postOrder(*w*)
  visit(*v*)



Binary Tree

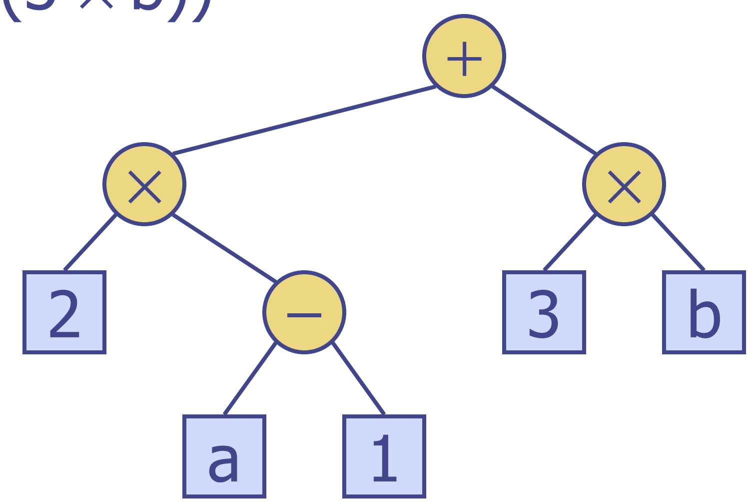
- A *binary tree* is a tree with the following properties:
  - Each internal node has two children
  - The children of a node are an ordered pair
- We call the children of an internal node *left child* and *right child*
- Alternative recursive definition: a binary tree is either
  - a tree consisting of a single node, or
  - a tree whose root has an ordered pair of children, each of which is a binary tree

- Applications:
  - arithmetic expressions
  - decision processes
  - searching



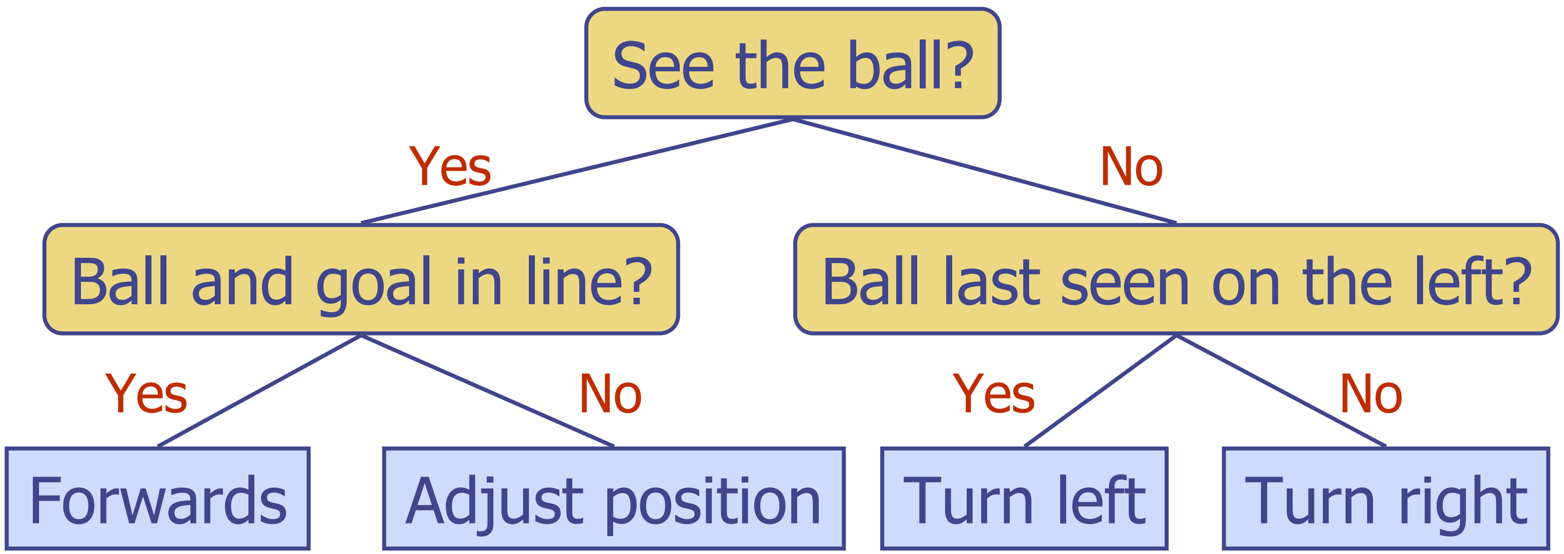
Arithmetic Expression Tree

- Binary tree associated with an arithmetic expression
  - internal nodes: operators
  - leaves: operands
- Example: arithmetic expression tree for the expression $(2 \times (a - 1) + (3 \times b))$



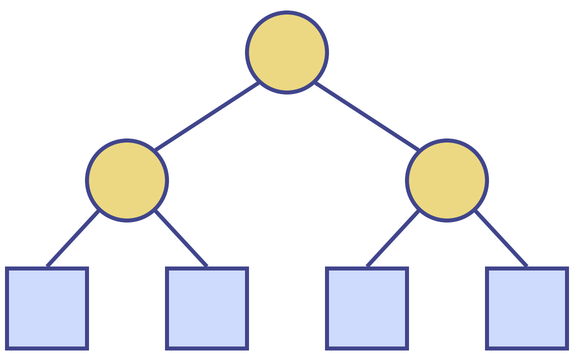
Decision Tree

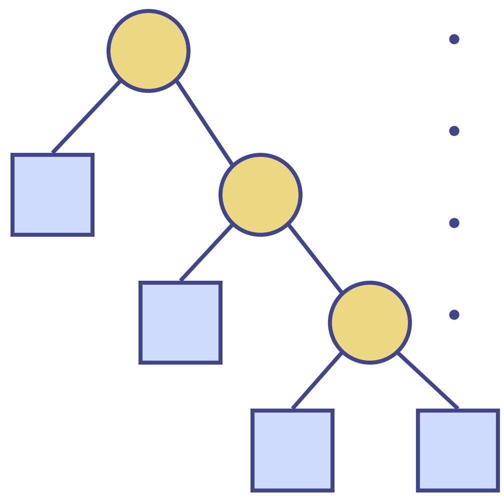
- Binary tree associated with a decision process
  - internal nodes: questions with yes/no answer
  - leaves: decisions
- Example: shooting (robots playing football)



Properties of Binary Trees

- Notation
  - n number of nodes
  - l number of leaves
  - i number of internal nodes
  - h height
- Properties:
  - $l = i + 1$
  - $n = 2l - 1$
  - $h \leq i$
  - $h \leq (n - 1)/2$
  - $l \leq 2^h$
  - $h \geq \log_2 l$
  - $h \geq \log_2 (n + 1) - 1$





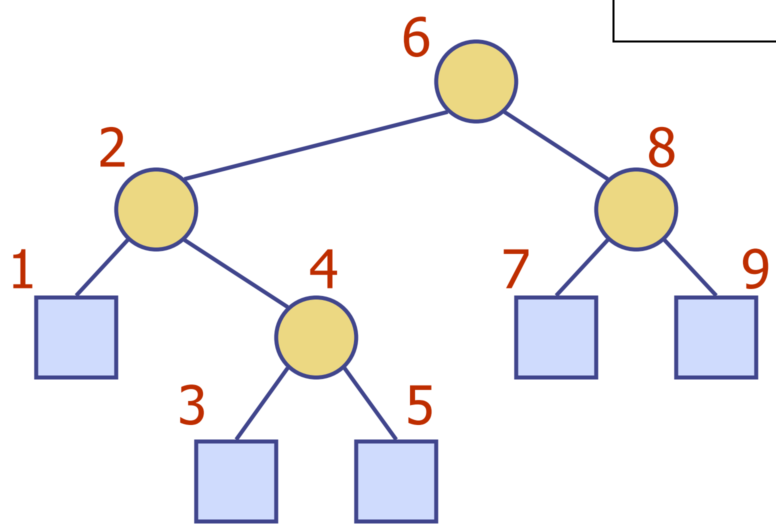
BinaryTree ADT

- The BinaryTree ADT extends the Tree ADT, i.e., it inherits all the methods of the Tree ADT
- Update methods may be defined by data structures implementing the BinaryTree ADT
- Additional methods:
  - position leftChild(p)
  - position rightChild(p)
  - position sibling(p)

Inorder Traversal

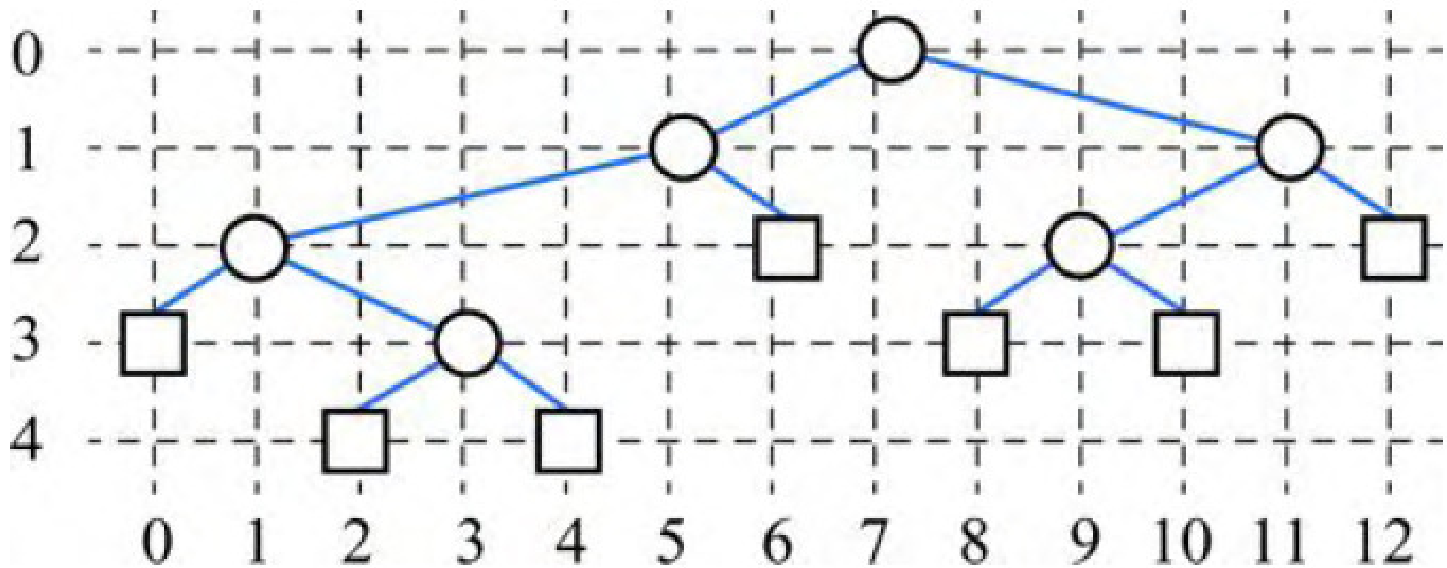
- In an *inorder traversal*, a node is visited after its left subtree and before its right subtree

```
Algorithm inOrder(v)
    if isInternal(v)
        inOrder(leftChild(v))
    visit(v)
    if isInternal(v)
        inOrder(rightChild(v))
```



Inorder Traversal – Application

- Application: draw a binary tree.
  Assign x- and y-coordinates to node v, where
  - $x(v)$ = inorder rank of v
  - $y(v)$ = depth of v



Exercise: Preorder & InOrder Traversal

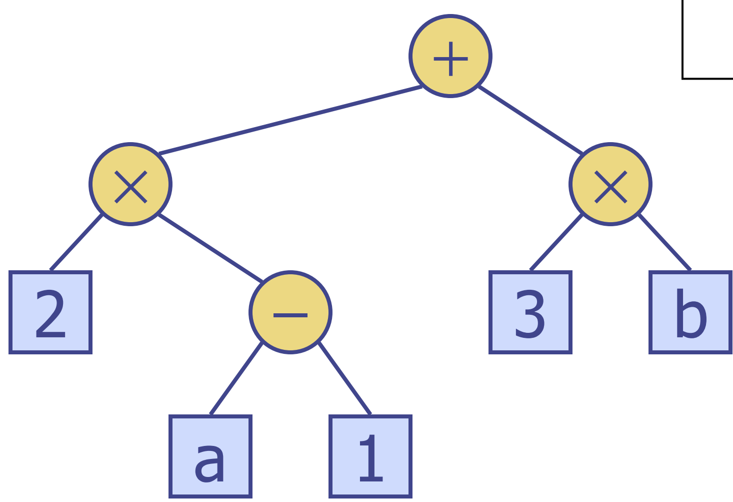
- Draw a (single) binary tree T, such that
  - Each internal node of T stores a single character
  - A preorder traversal of T yields EXAMFUN
  - An inorder traversal of T yields MAFXUEN

Print Arithmetic Expressions

Specialization of an inorder traversal

- print operand or operator when visiting node
- print “(“ before traversing left subtree
- print “)“ after traversing right subtree

Algorithm *printExpression*(*v*)
    if *hasLeft*(*v*)
        print (“(’’)
        printExpression(*leftChild*(*v*))
    print(*v.element*())
    if *hasRight*(*v*)
        printExpression(*rightChild*(*v*))
        print (“)’’)



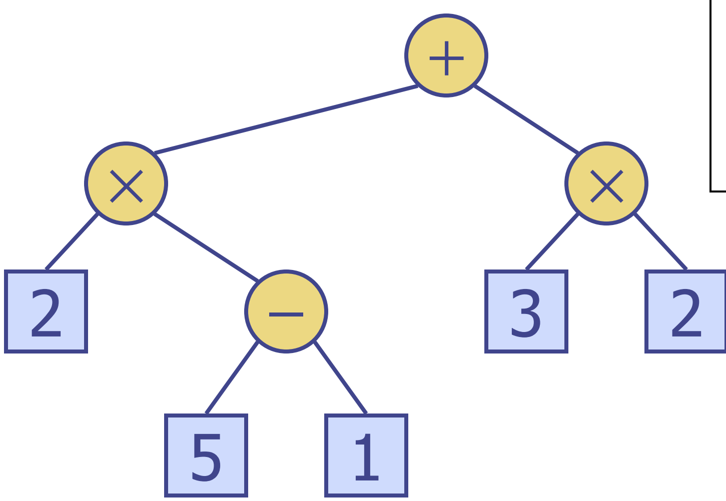
$$((2 \times (a - 1)) + (3 \times b))$$

Evaluate Arithmetic Expressions

Specialization of a postorder traversal

- recursive method returning the value of a subtree
- when visiting an internal node, combine the values of the subtrees

Algorithm *evalExpr*(*v*)
  if *isExternal*(*v*)
    return *v.element*()
  else
    x ← *evalExpr*(*leftChild*(*v*))
    y ← *evalExpr*(*rightChild*(*v*))
    ◊ ← operator stored at *v*
    return *x* ◊ *y*

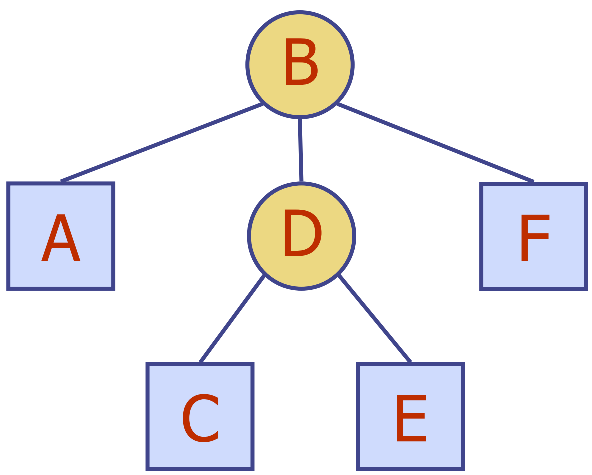


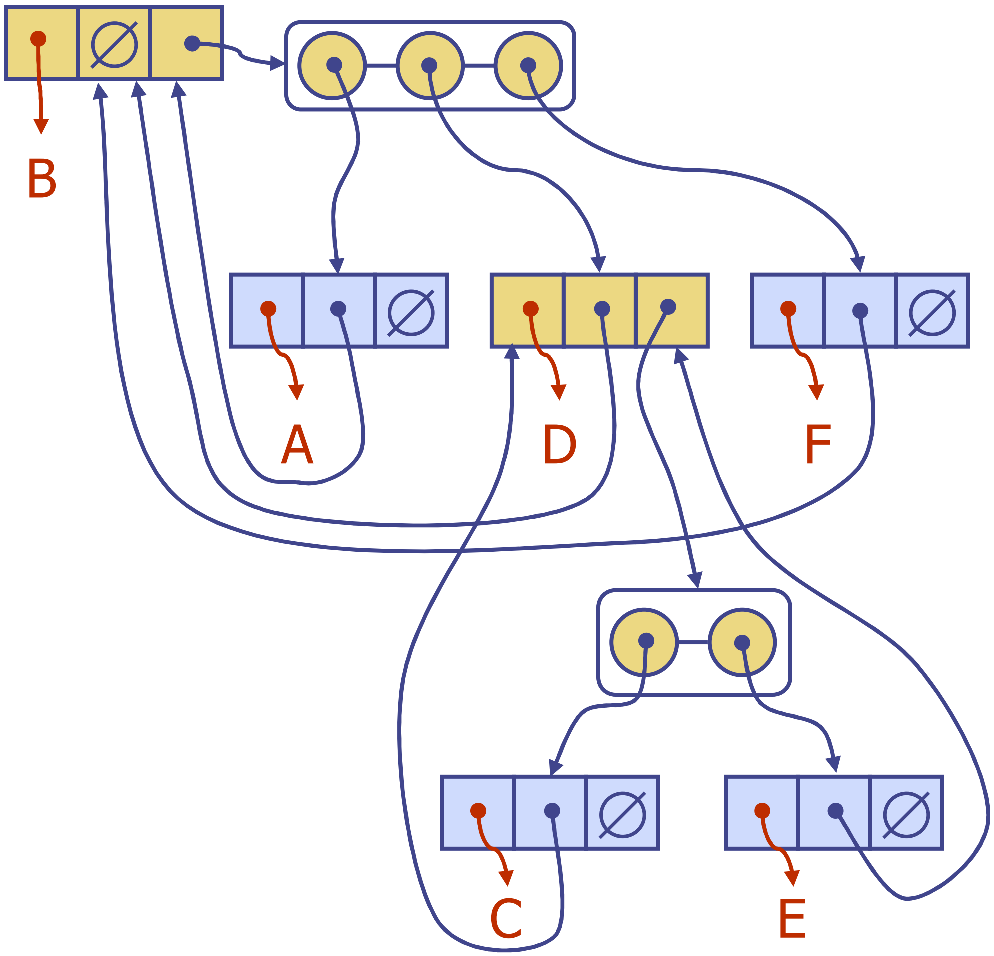
Exercise: Arithmetic Expressions

- Draw an expression tree that has
  - Four leaves, storing the values 1, 5, 6, and 7
  - 3 internal nodes, storing operations +, -, *, / (operators can be used more than once, but each internal node stores only one)
  - The value of the root is 21

Data Structure for Trees

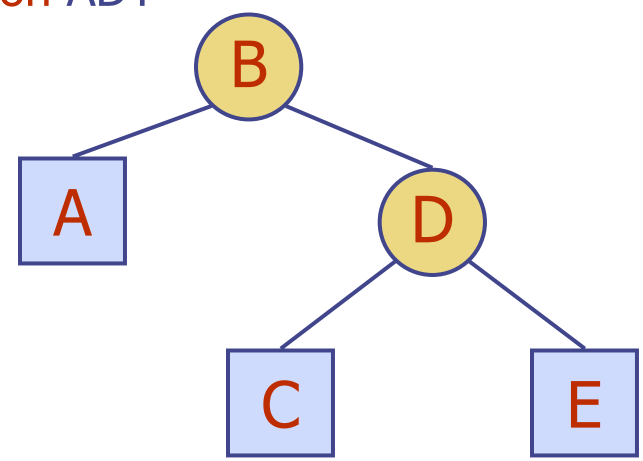
- A node is represented by an object storing
  - Element
  - Parent node
  - Sequence of children nodes
- Node objects implement the Position ADT

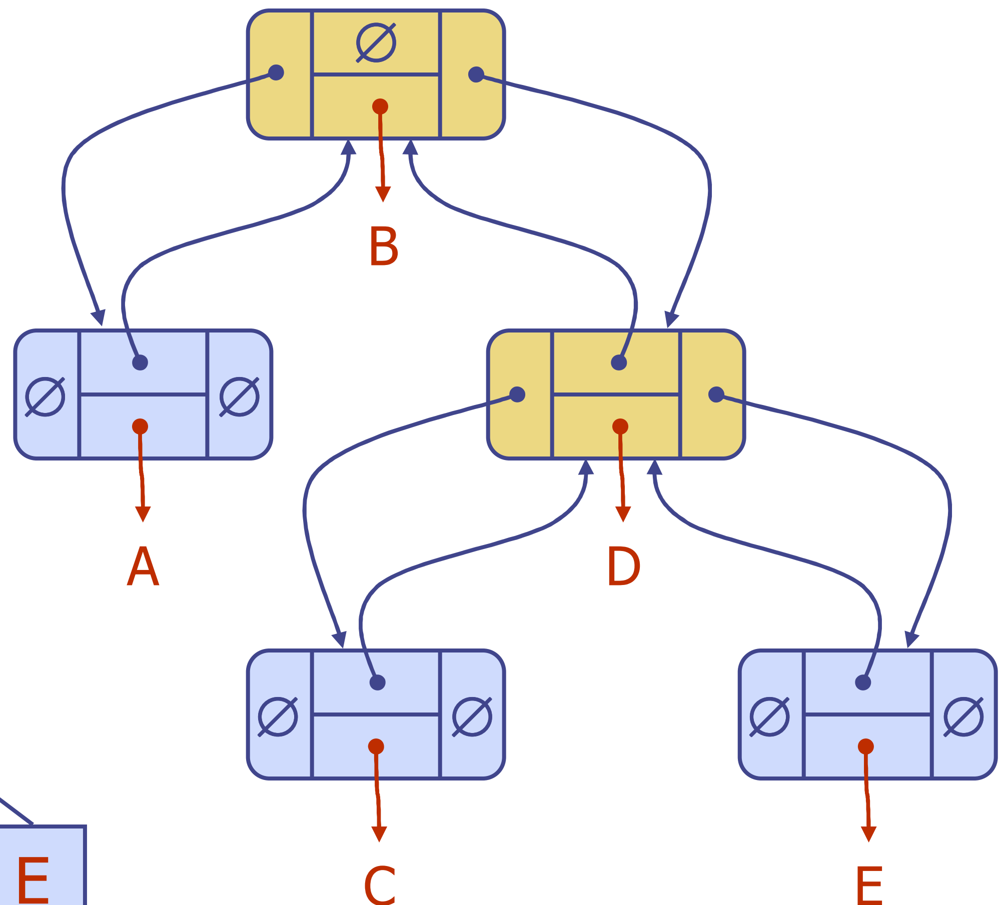




Data Structure for Binary Trees

- A node is represented by an object storing
  - Element
  - Parent node
  - Left child node
  - Right child node
- Node objects implement the Position ADT





C++ Implementation

- Tree interface
- BinaryTree interface extending Tree
- Classes implementing Tree and BinaryTree and providing
  - Constructors
  - Update methods
  - Print methods
- Examples of updates for binary trees
  - expandExternal(v)
  - removeAboveExternal(w)