Recursion

Data structures and Algorithms

What is recursion?

- A way of thinking about problems.
- A method for solving problems.
- Related to mathematical induction.

- In programming:
  - a function calls itself
    - direct recursion

```
int f () {
   ... f(); ...
}
```

  - a function calls its invoker
    - indirect recursion

```
int f () {
   ... g(); ...
}
```

```
int g () {
   ... f(); ...
}
```

Outline of a Recursive Function


FIGURE 7.1
A Set of Nested Wooden Figures

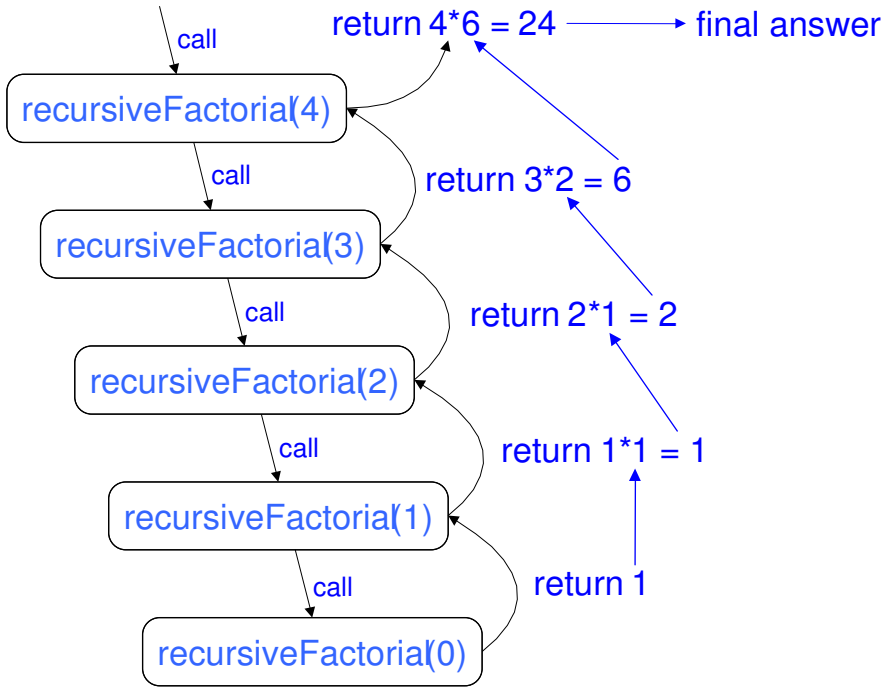
base case →

recursive case →

```
if (answer is known)
    provide the answer
else
    make a recursive call
    to solve a smaller version
    of the same problem
```

Recursive Factorial Method

- n! = n * (n-1) * (n-2) * … * 3 * 2 * 1
- n! = n * (n-1)!
- 0! = 1




```
Algorithm recursiveFactorial(n)
    if n==0 then
        return 1
    else
        return n * recursiveFactorial(n-1)
```

Fibonacci sequence

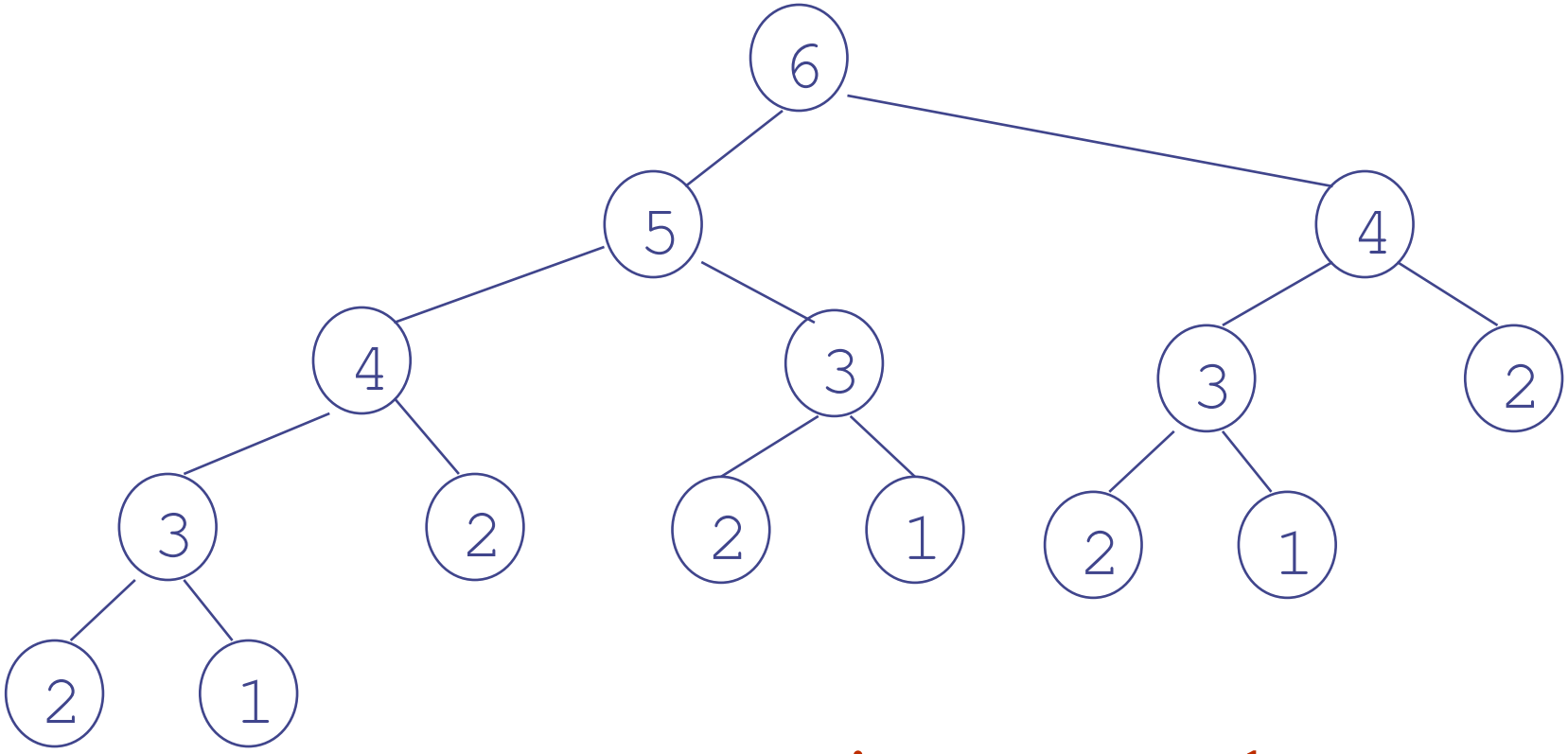
1, 1, 2, 3, 5, 8, 13, 21, ….

$$\text{fib}(n) = \begin{cases} 1 & \text{for } n == 1 \\ 1 & \text{for } n == 2 \\ \text{fib}(n-2) + \text{fib}(n-1) & \text{for } n > 2 \end{cases}$$

```
Algorithm fib(n)
    if n<=2 then
        return 1
    else
        return fib(n-2) + fib(n-1)
```

Tracing fib(6)

```
Algorithm fib(n)
    if n<=2 then
        return 1
    else
        return fib(n-2) + fib(n-1)
```



computation repeats!

Design a Recursive Algorithm

- There must be at least one case (the base case), for a small value of n, that can be solved directly
- A problem of a given size n can be split into one or more *smaller* versions of the *same* problem (recursive case)
- Recognize the base case and provide a solution to it
- Devise a strategy to split the problem into smaller versions of itself while making progress toward the base case
- Combine the solutions of the smaller problems in such a way as to solve the larger problem

Euclid's Algorithm

- Finds the greatest common divisor of two nonnegative integers that are not both 0
- Recursive definition of gcd algorithm
  - gcd (a, b) = a (if b is 0)
  - gcd (a, b) = gcd (b, a % b) (if b != 0)
- Implementation:

```
int gcd (int a, int b)
{
    if (b == 0)
        return a;
    else
        return gcd (b, a % b);
}
```

Iterative vs. recursive gcd

```
int gcd (int a, int b)
{
    int temp;
    while (b != 0)
    {
        temp = b;
        b = a % b;
        a  = temp;
    }
    return a;
}
```

```
int gcd (int a, int b)
{
    if (b == 0)
        return a;
    else
        return gcd (b, a % b);
}
```

Multiple recursion

- Tail recursion: a linearly recursive method makes its recursive call as its last step.
  - e.g. recursive gcd
  - Can be easily converted to non-recursive methods
- Binary recursion: there are **two** recursive calls for each non-base case
  - e.g. fibonaci sequence
- Multiple recursion: makes potentially **many** recursive calls (not just one or two).

Multiple recursion – Example

List all 'abc' strings of length l

```
void listAllStrings(int length, char* start)
{
    if (length < 1) { //base case: empty string
        *start = '\0';
        output();
    } else { //recursive case: reduce length by 1
        for (char c = 'a'; c <= 'c'; c++) {
            *start = c;
            listAllStrings(length-1, start+1);
        }
    }
}
```

```
aaa
aab
aac
aba
abb
abc
aca
acb
acc
baa
bab
…
ccc
```

Why using recursion?

- Recursion makes your code **faster? No!**
  - overhead for function call and return
  - values recomputed
- Recursion uses **less memory? No!**
  - overhead for a function call and return (stack memory)
- Recursion makes your code **simple? Sometimes.**
  - readable code that is easy to debug