Modal parameter estimation of ambient excited structures using time-frequency analysis based methods

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Contents

- Time-frequency analysis
- Closed frequencies extraction
- Modal estimation using wavelet transform
- Modal estimation using Hilbert-Huang transform

Objectives

1. How can wavelet transform extract closed natural frequencies???
2. What is Empirical Mode Decomposition (EMD)? and How is combination between EMD and Hilbert transform for modal parameter estimation of ambient excited structures
Introduction

- Modern system identification techniques using ambient data
- **Frequency Domain Decomposition** (Frequency domain)
- **Stochastic Subspace Identification** (Time domain)
- **Time-frequency analysis** (Time-frequency plane)
  - Wavelet transform-based method
  - Hilbert-Huang transform (Hilbert spectrum)-based method
Literature reviews

- Wavelet transform (WT) proposed by Daubechies (1992), applied for system identification by some authors. However, troublesome difficulties are such as:
  - Time-frequency resolutions in analysis
  - High modal parameters
  - Close natural frequencies
  - Many done with simulated signal, not real one with noises
  - Using traditional Morlet wavelet, not modified Morlet wavelet

- Hilbert-Huang transform (HHT) developed by Huang (1996), applied for many engineering topics, recently approaching to system identification.

- WT and HHT advantage such as:
  - Time-frequency information
  - Concept of time-dependant instantaneous quantities
  - Nonstationary, transient, nonlinear analysis

References:
- [Staszewski 1997; Ruzzene 1997; Ladies 2002&2007; Slavic 2003; Kijewsky 2003; Meo 2006; Chen 2009]
- [Huang 1999&2005; Yang 2004; Peng et al., 2005]
**Time-series analysis and resolution**

**Time Series**

\[ \Delta t = \frac{1}{f_s} \]

- \( f_s \): sampling rate (Hz)

**Short-time Fourier Transform**

\[ \Delta t \Delta f \geq \alpha \]

\[ \Delta t = \frac{f_s}{\sqrt{2}f} \]

\[ \Delta f = \frac{f}{2\sqrt{2}\pi f_s} \]

**Fourier Transform**

\[ \Delta f = \frac{1}{T} = \frac{f_s}{N} \]

- \( T \) : total interval (s)
- \( N \) : Number of samples

**Wavelet Transform**

\[ f_{max} = \frac{f_s}{n} \]

**Note:** Cannot obtain optimal time & frequency resolutions simultaneously
Wavelet transform

Wavelet transform coefficient

\[ W^X_{\psi}(s, \tau) = \frac{1}{\sqrt{s}} \int_{-\infty}^{+\infty} X(t) \psi^* \left( \frac{t-\tau}{s} \right) \, dt \]

Complex Morlet wavelet

\[ \psi(t) = (\pi f_b)^{-0.5} \exp(j2\pi f_c t) \exp(-t^2/2f_b) \]

Find solution as

\[ X(t) = \sum_{i=1}^{N} A_i \exp(-2\pi \zeta_i f_i t) \cos(2\pi f_{di} t + \theta_i) + \text{perturbation} \]

Wavelet coefficient at certain scale \( s=s_i=f_c/f_i \)

\[ W^X_{\psi}(s_i, t) = \frac{\sqrt{s_i}}{2} A_i \exp(-2\pi \zeta_i f_i t) \exp(j(2\pi f_{di} t + \theta_i)) \]

Analytic form

\[ W^X_{\psi}(s_i, t) = B_i(t) \exp(j\phi_i(t)) \]

where

\[ B_i(t) = (\sqrt{s_i}/2) A_i \exp(-2\pi \zeta_i f_i t) \]

\[ \phi_i(t) = 2\pi f_i \sqrt{1-\zeta_i^2} \, t + \theta_i \]

[Meo et al., 2006, Ladies at al., 2007]
Modified complex Morlet wavelet

Time-frequency resolution analysis

- Traditional complex Morlet wavelet
  - Frequency resolution: $\Delta f = \frac{f}{2\sqrt{2\pi f_c}}$
  - Time resolution: $\Delta t = \frac{f_c}{\sqrt{2f}}$

- Modified complex Morlet wavelet
  - Frequency resolution: $\Delta f = \frac{f}{2mf_c\sqrt{f_b}}$
  - Time resolution: $\Delta t = \frac{f_c\sqrt{f_b}}{2f}$

[Kijewski & Kareem, 2003]
[Meo et al., 2005]
[Ladies et al., 2007]
Hilbert transform

Hilbert transform (HT) is well-known signal analysis to build up analytic signal $Z(t)$ from real measured signal $X(t)$

\[ Z(t) = X(t) + iY(t) = A(t) \exp(\varphi(t)) \]

where \[ Z(t) = H[X(t)] \]

H[.] Hilbert transform

Instantaneous module

\[ A(t) = \sqrt{X(t)^2 + Y(t)^2} \]

Instantaneous phase

\[ \varphi(t) = \arctan(Y(t)/X(t)) \]

Instantaneous frequency and damping

\[ f(t) = \frac{1}{2\pi} \frac{d\varphi(t)}{dt} \]

\[ \frac{d \ln Z(t)}{dt} = -2\pi \zeta_i f_i \]

 Limitation of Hilbert transform is to apply for mono-component (can be converted to analytic form), but practical signal is multi-component one

Empirical Mode Decomposition (EMD) proposed by Huang 1996 is able to decompose signal into nearly mono-components called Intrinsic Mode Functions (IMFs), friendly used with HT

[Agneni and Crema, 1988]

[Zhang and Tamura, 2003]
EMD

[Intrinsic Mode Function] [Huang et al., 1996]

In the whole signal segment, the number of extrema and the number of zero crossing must be either equal or differ at most by one.
At any point, the mean value of the envelope defined by the local maxima and the envelope defined by the local minima is zero.

[Sifting process (Iterative algorithm)]
Find all maxima $M_i$ ($i=1,2,...$) and minima $m_j$ ($j=1,2,...$) from signal $x(t)$
Build upper envelope $M(t)=f_M(M_i,t)$ and lower envelope $m(t)=f_m(m_j,t)$ from maxima and minima (interpolating function using cubic spline)
Determine the mean line: $e_1(t)=(M(t)+m(t))/2$
Subtract mean line from the signal: $h_1(t)=x(t) - e_1(t)$
Iterate step 1 through step 4, stop when $h(t)$ unchanged and $e(t)$ is constant value. Obtain IMF1
EMD

\[ X(t) = \sum_{i=1}^{N} IMF_i(t) + R_d \]

[Huang, 2005]
Close natural frequencies extraction

Data

Note: Acceleration data from experimental one-storey building

PSD

\[
f = \frac{(3.59 + 3.77)}{2} = 3.68 \text{ Hz}
\]

\[
\Delta f = 3.77 - 3.59 = 0.18 \text{ Hz}
\]
Close natural frequencies extraction

Frequency Domain Decomposition

Spectral values (Singular values)

![Graph showing normalized spectral values and natural frequencies]
Close natural frequencies extraction

Stochastic Subspace Identification

**Stability diagram**

- $s=80$, $k=20$
- $s=100$, $k=20$

$s$: number of block rows in Hankel matrix

$k$: number of System orders in decomposition

**Band 3.5-3.9Hz**

- $f_1 = 3.59$ Hz
- $f_2 = 3.75$ Hz

**PSD**

- Band 3.5 - 3.9 Hz
- $f_1 = 3.59$ Hz
- $f_2 = 3.75$ Hz

- Frequency (Hz)
- Natural frequency (Hz)
- Number of poles
Close natural frequencies extraction

Wavelet transform coefficients

Traditional Morlet
Complex Morlet wavelet \([fc=1, fb=2]\)

Modified Morlet
Complex Morlet wavelet \([fc=2.5, fb=30]\)

Complex Morlet wavelet \([fc=3, fb=75]\)

Slide at \(\tau=140\)s

Fig. Frequency resolution analysis for close frequencies extraction
Close natural frequencies extraction

Wavelet transform coefficients

Complex Morlet wavelet $[fc=3, fb=75]$

Wavelet logarithmic amplitude for damping estimation

Damping estimation at $f_1$

Damping estimation at $f_2$

Fig. Damping estimation from wavelet logarithmic amplitude
# Close natural frequencies extraction

## Tentative discussion on feasibility for closed frequencies extraction

<table>
<thead>
<tr>
<th>Methods</th>
<th>Frequency Extraction</th>
<th>Damping Extraction</th>
</tr>
</thead>
<tbody>
<tr>
<td>FDD</td>
<td>✅ Very good High frequency resolution providing sampling rate and sampling time length</td>
<td>✅ Troublesome difficulty Mode shape info required</td>
</tr>
<tr>
<td>SSI</td>
<td>✅ Possible Troublesome parameters: numbers of block rows; number of system orders</td>
<td>✅ Possible Noise can be removed Further investigation</td>
</tr>
<tr>
<td>WT</td>
<td>✅ Possible Frequency resolution analysis required</td>
<td>✅ Possible Noise can be removed Further investigation</td>
</tr>
</tbody>
</table>

**Time & memory requirements:**

Frequency resolution???

Selected time interval???
Wavelet transform-based technique

Data

Note: Displacement at 1\textsuperscript{nd} floor of five-storey building

PSD

Natural frequencies

<table>
<thead>
<tr>
<th>Mode</th>
<th>Frequency</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mode 1</td>
<td>1.74Hz</td>
</tr>
<tr>
<td>Mode 2</td>
<td>5.35Hz</td>
</tr>
<tr>
<td>Mode 3</td>
<td>8.84Hz</td>
</tr>
<tr>
<td>Mode 4</td>
<td>13.68Hz</td>
</tr>
<tr>
<td>Mode 5</td>
<td>18.23Hz</td>
</tr>
</tbody>
</table>
Wavelet transform-based technique

Wavelet transform coefficients

Bandwidth 0-20Hz, $f_c=2, f_b=20$

- High modes 3rd, 4th, 5th not observed
- Resolution adjustment must be required for high modes with low energy levels
Wavelet transform-based technique

- Frequency resolution adjustment techniques
  - Bandwidth resolution adjustment
    Bandwidths 0-5Hz; 5-10Hz; 8-12Hz; 12-16Hz; 16-20Hz
    Note: Wavelet transform with different resolutions at each bandwidth

  - Bandpass filtering adjustment
    Band1: 0-3Hz; Band2: 3-6Hz; Band3: 6-12Hz; Band4: 12-24Hz; Band5: 24-50Hz
    Note: First, bandpass filtering, then wavelet transform

  - Narrowed filtering adjustment around natural frequencies $f_1$, $f_2$, $f_3$, $f_4$, $f_5$
    Note: First, narrowed filtering around natural frequencies, then wavelet transform
Wavelet transform-based technique

Wavelet transform coefficients

**Bandwidth 0-5Hz / fc=2, fb=20**

**Bandwidth 5-10Hz / fc=2, fb=50**

**Bandwidth 8-12Hz / fc=3, fb=75**

**Bandwidth 12-16Hz / fc=5, fb=50**

**Natural frequencies**
- Mode 1: 1.74Hz
- Mode 2: 5.32Hz
- Mode 3: 8.81Hz
- Mode 4: 13.64Hz
- Mode 5: 18.07Hz

Fig. Bandwidth resolution adjustment
Wavelet transform-based technique

Wavelet transform coefficients

**Band 1 0-3 Hz** / $f_c=2$, $f_b=20$

**Band 2 3-6 Hz** / $f_c=2$, $f_b=20$

**Band 3 6-12 Hz** / $f_c=2$, $f_b=20$

**Band 4 12-24 Hz** / $f_c=5$, $f_b=50$

---

**Natural frequencies**

- **Mode 1**: 1.73 Hz
- **Mode 2**: 5.34 Hz
- **Mode 3**: 8.82 Hz
- **Mode 4**: 13.59 Hz
- **Mode 5**: 18.0 Hz

---

**Fig. Bandpass filtering adjustment**
Wavelet transform-based technique

Filtering at $f_1 / f_c = 1$, $f_b = 20$

Filtering at $f_2 / f_c = 2$, $f_b = 20$

Filtering at $f_3 / f_c = 2$, $f_b = 20$

Filtering at $f_4 / f_c = 4$, $f_b = 50$

Natural frequencies

<table>
<thead>
<tr>
<th>Mode</th>
<th>Frequency (Hz)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mode 1</td>
<td>1.73Hz</td>
</tr>
<tr>
<td>Mode 2</td>
<td>5.35Hz</td>
</tr>
<tr>
<td>Mode 3</td>
<td>8.83Hz</td>
</tr>
<tr>
<td>Mode 4</td>
<td>13.64Hz</td>
</tr>
<tr>
<td>Mode 5</td>
<td>18.04Hz</td>
</tr>
</tbody>
</table>

Fig. Narrowed filtering adjustment
Wavelet transform-based technique

Wavelet logarithmic amplitude envelope

**Linear fitting for damping of 1\textsuperscript{st} mode**

Wavelet logarithmic amplitude at $f_1=1.73\text{Hz}$

![Graph showing linear fitting for damping of 1\textsuperscript{st} mode]

$y = -0.033x - 5.7$

Damping ratios
- Mode 1: 0.52%
- Mode 2: 0.52%

**Linear fitting for damping ratio of 2\textsuperscript{nd} mode**

Wavelet logarithmic amplitude at $f_2=5.35\text{Hz}$

![Graph showing linear fitting for damping ratio of 2\textsuperscript{nd} mode]

$y = -0.13x - 8.6$

Damping ratios
- Mode 3: 0.52%
- Mode 4: 2.07%
- Mode 5: 2.07%

**Linear fitting for damping of 3\textsuperscript{rd} mode**

Wavelet logarithmic amplitude at $f_3=8.83\text{Hz}$

![Graph showing linear fitting for damping of 3\textsuperscript{rd} mode]

$y = -0.13x - 2.2$

Damping ratios
- Mode 1: 2.07%
- Mode 2: 2.07%
- Mode 4: 1.75%
- Mode 5: 2.22%

**Linear fitting for damping ratio of 4\textsuperscript{th} mode**

Wavelet logarithmic amplitude at $f_4=13.64\text{Hz}$

![Graph showing linear fitting for damping ratio of 4\textsuperscript{th} mode]

$y = -0.11x - 8$

Damping ratios
- Mode 3: 2.07%

Fig. Damping estimation from wavelet logarithmic amplitude envelope
Hilbert-Huang transform-based technique

Intrinsic Mode Functions (IMFs)

\[
\text{IMF1} \div \text{IMF4} \quad \text{IMF5} \div \text{IMF9}
\]

Data (20 ÷ 30s)

IMF1 (20 ÷ 30s)

IMF2 (20 ÷ 30s)
Hilbert-Huang transform-based technique

**IMF1 ÷ IMF4**

**PSD**

**IMF5 ÷ IMF9**
Instantaneous modules
Instantaneous frequency (in IMF1 ÷ 8)
Hilbert spectrum

Hilbert spectrum of IMF1

Hilbert spectrum of IMF2

[Smoothing & Embedding instantaneous frequency in Hilbert spectrum]
Further works

- Modal parameter estimation techniques for close natural frequency structures
- WT and HHT techniques for system identification
- Empirical Mode Decomposition applies for unsteady pressure fields to understand physical meaning
Thank you for your attention