1.9 Exercises

1. Evaluate the following functions:
   
   a. \( \sin t \delta(t - \frac{\pi}{6}) \)
   
   b. \( \cos 2t \delta(t - \frac{\pi}{4}) \)
   
   c. \( \cos^2 t \delta(t - \frac{\pi}{2}) \)
   
   d. \( \tan 2t \delta(t - \frac{\pi}{8}) \)
   
   e. \( \int_{-\infty}^{\infty} t^2 e^{-t} \delta(t - 2) dt \)
   
   f. \( \sin^2 t \delta(t - \frac{\pi}{2}) \)

2.

   a. Express the voltage waveform \( v(t) \) shown in Figure 1.24, as a sum of unit step functions for the time interval \( 0 < t < 7 \) s.

   b. Using the result of part (a), compute the derivative of \( v(t) \), and sketch its waveform.

   ![Figure 1.24. Waveform for Exercise 2](image-url)
2.6 Exercises

1. Find the Laplace transform of the following time domain functions:
   a. $12$
   b. $6u_0(t)$
   c. $24u_0(t - 12)$
   d. $5tu_0(t)$
   e. $4t^5u_0(t)$

2. Find the Laplace transform of the following time domain functions:
   a. $j8$
   b. $j5 \angle -90^\circ$
   c. $5e^{-5t}u_0(t)$
   d. $8t^7e^{-5t}u_0(t)$
   e. $15\delta(t - 4)$

3. Find the Laplace transform of the following time domain functions:
   a. $(t^3 + 3t^2 + 4t + 3)u_0(t)$
   b. $3(2t - 3)\delta(t - 3)$
   c. $(3\sin 5t)u_0(t)$
   d. $(5\cos 3t)u_0(t)$
   e. $(2\tan 4t)u_0(t)$ Be careful with this! Comment and skip derivation.

4. Find the Laplace transform of the following time domain functions:
   a. $3t(\sin 5t)u_0(t)$
   b. $2t^2(\cos 3t)u_0(t)$
   c. $2e^{-5t}\sin 5t$
5. Find the Laplace transform of the following time domain functions:
   a. $5t u_0(t - 3)$
   b. $(2t^2 - 5t + 4)u_0(t - 3)$
   c. $(t - 3)e^{-2t}u_0(t - 2)$
   d. $(2t - 4)e^{2(t-2)}u_0(t - 3)$
   e. $4te^{-3t}(\cos 2t)u_0(t)$

6. Find the Laplace transform of the following time domain functions:
   a. $\frac{d}{dt}(\sin 3t)$
   b. $\frac{d}{dt}(3e^{-4t})$
   c. $\frac{d}{dt}(t^2 \cos 2t)$
   d. $\frac{d}{dt}(e^{-2t}\sin 2t)$
   e. $\frac{d}{dt}(t^2e^{-2t})$

7. Find the Laplace transform of the following time domain functions:
   a. $\frac{\sin t}{t}$
   b. $\int_0^t \frac{\sin \tau}{\tau} d\tau$
   c. $\frac{\sin at}{t}$
   d. $\int_t^\infty \frac{\cos \tau}{\tau} d\tau$
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e. \int_{t}^{\infty} \frac{e^{-\tau}}{\tau} d\tau

8. Find the Laplace transform for the sawtooth waveform \( f_{ST}(t) \) of Figure 2.8.

9. Find the Laplace transform for the full rectification waveform \( f_{FR}(t) \) of Figure 2.9.
### 3.6 Exercises

1. Find the Inverse Laplace transform of the following:

   a. \( \frac{4}{s + 3} \)
   
   b. \( \frac{4}{(s + 3)^2} \)
   
   c. \( \frac{4}{(s + 3)^4} \)
   
   d. \( \frac{3s + 4}{(s + 3)^5} \)
   
   e. \( \frac{s^2 + 6s + 3}{(s + 3)^5} \)

2. Find the Inverse Laplace transform of the following:

   a. \( \frac{3s + 4}{s^2 + 4s + 85} \)
   
   b. \( \frac{4s + 5}{s^2 + 5s + 18.5} \)
   
   c. \( \frac{s^2 + 3s + 2}{s^3 + 5s^2 + 10.5s + 9} \)
   
   d. \( \frac{s^2 - 16}{s^3 + 8s^2 + 24s + 32} \)
   
   e. \( \frac{s + 1}{s^3 + 6s^2 + 11s + 6} \)

3. Find the Inverse Laplace transform of the following:

   a. \( \frac{3s + 2}{s^2 + 25} \)
   
   b. \( \frac{5s^2 + 3}{(s^2 + 4)^2} \) (See hint on next page)
**Exercises**

**Hint:**

\[
\begin{align*}
\frac{1}{2\alpha} (\sin \alpha t + \alpha t \cos \alpha t) & \Leftrightarrow \frac{s^2}{(s^2 + \alpha^2)^2} \\
\frac{1}{2\alpha} (\sin \alpha t - \alpha t \cos \alpha t) & \Leftrightarrow \frac{1}{(s^2 + \alpha^2)^2}
\end{align*}
\]

c. \(\frac{2s + 3}{s^2 + 4.25s + 1}\)

d. \(\frac{s^3 + 8s^2 + 24s + 32}{s^2 + 6s + 8}\)

e. \(e^{-2s} \frac{3}{(2s + 3)^3}\)

4. Use the Initial Value Theorem to find \(f(0)\) given that the Laplace transform of \(f(t)\) is

\[\frac{2s + 3}{s^2 + 4.25s + 1}\]

Compare your answer with that of Exercise 3(c).

5. It is known that the Laplace transform \(F(s)\) has two distinct poles, one at \(s = 0\), the other at \(s = -1\). It also has a single zero at \(s = 1\), and we know that \(\lim_{t \to \infty} f(t) = 10\). Find \(F(s)\) and \(f(t)\).
4.6 Exercises

1. In the circuit of Figure 4.22, switch $S$ has been closed for a long time, and opens at $t = 0$. Use the Laplace transform method to compute $i_L(t)$ for $t > 0$.

![Figure 4.22. Circuit for Exercise 1](image)

2. In the circuit of Figure 4.23, switch $S$ has been closed for a long time, and opens at $t = 0$. Use the Laplace transform method to compute $v_c(t)$ for $t > 0$.

![Figure 4.23. Circuit for Exercise 2](image)

3. Use mesh analysis and the Laplace transform method, to compute $i_1(t)$ and $i_2(t)$ for the circuit of Figure 4.24, given that $i_L(0^-) = 0$ and $v_c(0^-) = 0$.

![Figure 4.24. Circuit for Exercise 3](image)
4. For the \textit{s-domain} circuit of Figure 4.25,

a. compute the admittance $Y(s) = \frac{I_1(s)}{V_1(s)}$

b. compute the \textit{t-domain} value of $i_1(t)$ when $v_1(t) = u_0(t)$, and all initial conditions are zero.

\begin{figure}
\centering
\includegraphics[width=0.5\textwidth]{fig4.25.png}
\caption{Circuit for Exercise 4}
\end{figure}

5. Derive the transfer functions for the networks (a) and (b) of Figure 4.26.

\begin{figure}
\centering
\includegraphics[width=0.5\textwidth]{fig4.26.png}
\caption{Networks for Exercise 5}
\end{figure}

6. Derive the transfer functions for the networks (a) and (b) of Figure 4.27.

\begin{figure}
\centering
\includegraphics[width=0.5\textwidth]{fig4.27.png}
\caption{Networks for Exercise 6}
\end{figure}

7. Derive the transfer functions for the networks (a) and (b) of Figure 4.28.
8. Derive the transfer function for the networks (a) and (b) of Figure 4.29.

9. Derive the transfer function for the network of Figure 4.30. Using MATLAB, plot $|G(s)|$ versus frequency in Hertz, on a semilog scale.

$$R_1 = 11.3 \, \text{k}\Omega$$
$$R_2 = 22.6 \, \text{k}\Omega$$
$$R_3 = R_4 = 68.1 \, \text{k}\Omega$$
$$C_1 = C_2 = 0.01 \, \text{µF}$$
6.7 Exercises

1. Compute the impulse response \( h(t) = i_L(t) \) in terms of \( R \) and \( L \) for the circuit of Figure 6.36. Then, compute the voltage \( v_L(t) \) across the inductor.

\[
\begin{align*}
\delta(t) & \quad R \\
+ & \quad i_L(t) \\
\downarrow & \quad L
\end{align*}
\]

Figure 6.36. Circuit for Exercise 1

2. Repeat Example 6.4 by forming \( h(t - \tau) \) instead of \( u(t - \tau) \), that is, use the convolution integral

\[
\int_{-\infty}^{\infty} u(\tau)h(t - \tau)d\tau
\]

3. Repeat Example 6.5 by forming \( h(t - \tau) \) instead of \( u(t - \tau) \).

4. Compute \( v_1(t) * v_2(t) \) given that

\[
v_1(t) = \begin{cases} 
0 & t < 0 \\
4t & t \geq 0 
\end{cases}
\quad v_2(t) = \begin{cases} 
e^{-2t} & t \geq 0 \\
0 & t < 0
\end{cases}
\]

5. For the series \( RL \) circuit shown in Figure 6.37, the response is the current \( i_L(t) \). Use the convolution integral to find the response when the input is the unit step \( u_0(t) \).

\[
\begin{align*}
R & \quad \downarrow \\
\downarrow & \quad 1 \Omega \\
\downarrow & \quad i_L(t) \\
\downarrow & \quad L \quad 1 H
\end{align*}
\]

Figure 6.37. Circuit for Exercise 5

6. Compute \( v_{out}(t) \) for the network of Figure 6.38 using the convolution integral, given that \( v_{in}(t) = u_0(t) - u_0(t - 1) \).

\[
\begin{align*}
R & \quad \downarrow \\
\downarrow & \quad 1 \Omega \\
\downarrow & \quad 1 H \\
\downarrow & \quad i_L(t) \\
\downarrow & \quad i(t)
\end{align*}
\]
7. Compute $v_{out}(t)$ for the circuit of Figure 6.39 given that $v_{in}(t) = u_0(t) - u_0(t-1)$.

Hint: Use the result of Exercise 6.
7.14 Exercises

1. Compute the first 5 components of the trigonometric Fourier series for the waveform of Figure 7.47. Assume $\omega = 1$.

![Figure 7.47. Waveform for Exercise 1](image)

2. Compute the first 5 components of the trigonometric Fourier series for the waveform of Figure 7.48. Assume $\omega = 1$.

![Figure 7.48. Waveform for Exercise 2](image)

3. Compute the first 5 components of the exponential Fourier series for the waveform of Figure 7.49. Assume $\omega = 1$.

![Figure 7.49. Waveform for Exercise 3](image)

4. Compute the first 5 components of the exponential Fourier series for the waveform of Figure 7.50. Assume $\omega = 1$.

![Figure 7.50. Waveform for Exercise 4](image)
5. Compute the first 5 components of the exponential Fourier series for the waveform of Figure 7.51. Assume $\omega = 1$.

$$f(t)$$

![Figure 7.51. Waveform for Exercise 5](image1)

6. Compute the first 5 components of the exponential Fourier series for the waveform of Figure 7.52. Assume $\omega = 1$.

$$f(t)$$

![Figure 7.52. Waveform for Exercise 6](image2)
8.10 Exercises

1. Show that

\[ \int_{-\infty}^{\infty} u_\delta(t) \delta(t) dt = 1/2 \]

2. Compute

\[ \mathcal{F}\{te^{-at}u_\delta(t)\} \quad a > 0 \]

3. Sketch the time and frequency waveforms of

\[ f(t) = \cos \omega_0 t [u_\delta(t + T) - u_\delta(t - T)] \]

4. Derive the Fourier transform of

\[ f(t) = A[u_\delta(t + 3T) - u_\delta(t + T) + u_\delta(t - T) - u_\delta(t - 3T)] \]

5. Derive the Fourier transform of

\[ f(t) = \frac{A}{T}[u_\delta(t + T) - u_\delta(t - T)] \]

6. Derive the Fourier transform of

\[ f(t) = \left(\frac{A}{T} t + A\right)[u_\delta(t + T) - u_\delta(t)] + \left(-\frac{A}{T} t + A\right)[u_\delta(t) - u_\delta(t - \frac{T}{2})] \]

7. For the circuit of Figure 8.21, use the Fourier transform method to compute \( v_C(t) \).

![Circuit for Exercise 7](image)

\[ v_{in}(t) = 50 \cos 4tu_\delta(t) \]

Figure 8.21. Circuit for Exercise 7

8. The input-output relationship in a certain network is

\[ \frac{d^2 v_{out}(t)}{dt^2} + 5 \frac{dv_{out}(t)}{dt} + 6v_{out}(t) = 10v_{in}(t) \]

Use the Fourier transform method to compute \( v_{out}(t) \) given that \( v_{in}(t) = 2e^{-t}u_\delta(t) \).
9. In a bandpass filter, the lower and upper cutoff frequencies are \( f_1 = 2 \text{ Hz} \), and \( f_2 = 6 \text{ Hz} \) respectively. Compute the 1 \( \Omega \) energy of the input, and the percentage that appears at the output, if the input signal is \( v_{\text{in}}(t) = 3e^{-2t}u_0(t) \) volts.

10. In Example 8.4, we derived the Fourier transform pair

\[
A[u_0(t + T) - u_0(t - T)] \Leftrightarrow 2AT \frac{\sin\omega T}{\omega T}
\]

![Figure 8.22. Figure for Exercise 10](image)

Compute the percentage of the 1 \( \Omega \) energy of \( f(t) \) contained in the interval \(-\pi/T \leq \omega \leq \pi/T\) of \( F(\omega) \).
9.10 Exercises

1. Find the $Z$ transform of the discrete time pulse $p[n]$ defined as

$$p[n] = \begin{cases} 1 & n = 0, 1, 2, \ldots, m-1 \\ 0 & otherwise \end{cases}$$

2. Find the $Z$ transform of $a^n p[n]$ where $p[n]$ is defined as in Exercise 1.

3. Prove the following $Z$ transform pairs:
   a. $\delta[n] \Leftrightarrow 1$
   b. $\delta[n-1] \Leftrightarrow z^{-m}$
   c. $na^n u_0[n] \Leftrightarrow \frac{az}{(z-a)^2}$
   d. $n^2 a^n u_0[n] \Leftrightarrow \frac{az(z+a)}{(z-a)^3}$
   e. $[n+1]u_0[n] \Leftrightarrow \frac{z^2}{(z-1)^2}$

4. Use the partial fraction expansion to find $f[n] = Z^{-1}[F(z)]$ given that

$$F(z) = \frac{A}{(1-z^{-1})(1-0.5z^{-1})}$$

5. Use the partial fraction expansion method to compute the Inverse $Z$ transform of

$$F(z) = \frac{z^2}{(z+1)(z-0.75)^2}$$

6. Use the Inversion Integral to compute the Inverse $Z$ transform of

$$F(z) = \frac{1 + 2z^{-1} + z^{-3}}{(1-z^{-1})(1-0.5z^{-1})}$$

7. Use the long division method to compute the first 5 terms of the discrete time sequence whose $Z$ transform is

$$F(z) = \frac{z^{-1} + z^{-2} - z^{-3}}{1 + z^{-1} + z^{-2} + 4z^{-3}}$$
8. a. Compute the transfer function of the difference equation

\[ y[n] - y[n-1] = Tx[n-1] \]

b. Compute the response \( y[n] \) when the input is \( x[n] = e^{-naT} \)

9. Given the difference equation

\[ y[n] - y[n-1] = \frac{T}{2} \{ x[n] + x[n-1] \} \]

a. Compute the discrete transfer function \( H(z) \)

b. Compute the response to the input \( x[n] = e^{-naT} \)

10. A discrete time system is described by the difference equation

\[ y[n] + y[n-1] = x[n] \]

where

\[ y[n] = 0 \text{ for } n < 0 \]

a. Compute the transfer function \( H(z) \)

b. Compute the impulse response \( h[n] \)

c. Compute the response when the input is \( x[n] = 10 \text{ for } n \geq 0 \)

11. Given the discrete transfer function

\[ H(z) = \frac{z + 2}{8z^2 - 2z - 3} \]

write the difference equation that relates the output \( y[n] \) to the input \( x[n] \).
10.8 Exercises


2. A square waveform is represented by the discrete time sequence


Use MATLAB to compute and plot the magnitude $|X[m]|$ of this sequence.

3. Prove that

a. $x[n] \cos \frac{2\pi kn}{N} \iff \frac{1}{2} \{X[m-k] + X[m+k]\}$

b. $x[n] \sin \frac{2\pi kn}{N} \iff \frac{1}{j2} \{X[m-k] + X[m+k]\}$

4. The signal flow graph of Figure 10.6 is a decimation in time, natural-input, shuffled-output type FFT algorithm. Using this graph and relation (10.69), compute the frequency component $X[3]$. Verify that this is the same as that found in Example 10.5.

5. The signal flow graph of Figure 10.7 is a decimation in frequency, natural input, shuffled output type FFT algorithm. There are two equations that relate successive columns. The first is

$$Y_{dash}(R, C) = Y_{dash}(R_i, C-1) + Y_{dash}(R_j, C-1)$$

and it is used with the nodes where two dashed lines terminate on them.
The second equation is

\[ Y_{sol}(R, C) = W^m [Y_{sol}(R_i, C - 1) - Y_{sol}(R_j, C - 1)] \]

and it is used with the nodes where two solid lines terminate on them. The number inside the circles denote the power of \( W_N \), and the minus (\(-\)) sign below serves as a reminder that the bracketed term of the second equation involves a subtraction. Using this graph and the above equations, compute the frequency component \( X[3] \). Verify that this is the same as in Example 10.5.

![Signal flow graph for Exercise 5](image-url)
11.10 Exercises

1. The circuit of Figure 11.39 is a VCVS second-order high-pass filter whose transfer function is

\[
G(s) = \frac{V_{out}(s)}{V_{in}(s)} = \frac{Ks^2}{s^2 + (a/b)\omega_c s + (1/b)\omega_c^2}
\]

and for given values of \(a\), \(b\), and desired cutoff frequency \(\omega_c\), we can calculate the values of \(C_1, C_2, R_1, R_2, R_3\), and \(R_4\) to achieve the desired cutoff frequency \(\omega_c\).

For this circuit,

\[
R_2 = \frac{4b}{C_1 \left( a + \sqrt{a^2 + 8b(K - 1)} \right) \omega_c}
\]

\[
R_1 = \frac{b}{C_1^2 R_2 \omega_c^2}
\]

\[
R_3 = \frac{KR_2}{K - 1}, \quad K \neq 1
\]

\[
R_4 = KR_2
\]

and the gain \(K\) is

\[
K = 1 + \frac{R_4}{R_3}
\]

Using these relations, compute the appropriate values of the resistors to achieve the cutoff frequency \(f_c = 1\) KHz. Choose the capacitors as \(C_1 = 10/f_c \mu F\) and \(C_2 = C_1\). Plot \(|G(s)|\) versus frequency.

Solution using MATLAB is highly recommended.
2. The circuit of Figure 11.40 is a VCVS second-order band-pass filter whose transfer function is

\[ G(s) = \frac{V_{out}(s)}{V_{in}(s)} = \frac{K[ BW]s}{s^2 + [ BW]s + \omega_0^2} \]

![Figure 11.40. Circuit for Exercise 2](image)

Let \( \omega_0 = \text{center frequency} \), \( \omega_2 = \text{upper cutoff frequency} \), \( \omega_1 = \text{lower cutoff frequency} \), Bandwidth \( BW = \omega_2 - \omega_1 \), and Quality Factor \( Q = \omega_0 / BW \).

We can calculate the values of \( C_1, C_2, R_1, R_2, R_3, \) and \( R_4 \) to achieve the desired centered frequency \( \omega_0 \) and bandwidth \( BW \). For this circuit,

\[
\begin{align*}
R_1 &= \frac{2Q}{C_1\omega_0K} \\
R_2 &= \frac{2Q}{C_1\omega_0 \left\{ -1 + \sqrt{(K-1)^2 + 8Q^2} \right\}} \\
R_3 &= \frac{1}{C_1\omega_0^2} \left( \frac{1}{R_1} + \frac{1}{R_2} \right) \\
R_4 &= R_5 = 2R_3
\end{align*}
\]

Using these relations, compute the appropriate values of the resistors to achieve center frequency \( f_0 = 1 \text{ KHz} \), Gain \( K = 10 \), and \( Q = 10 \).

Choose the capacitors as \( C_1 = C_2 = 0.1 \text{ \(\mu\)F} \). Plot \(|G(s)| \) versus frequency.

Solution using MATLAB is highly recommended.
3. The circuit of Figure 11.41 is a VCVS second-order band elimination filter whose transfer function is

\[ G(s) = \frac{V_{out}(s)}{V_{in}(s)} = \frac{K(s^2 + \omega_0^2)}{s^2 + [BW]s + \omega_0^2} \]

Let \( \omega_0 \) = center frequency, \( \omega_2 \) = upper cutoff frequency, \( \omega_1 \) = lower cutoff frequency, Bandwidth \( BW = \omega_2 - \omega_1 \), Quality Factor \( Q = \omega_0/BW \), and gain \( K = 1 \).

We can calculate the values of \( C_1, C_2, R_1, R_2, R_3 \), and \( R_4 \) to achieve the desired centered frequency \( \omega_0 \) and bandwidth \( BW \). For this circuit,

\[
R_1 = \frac{1}{2\omega_0QC_1} \\
R_2 = \frac{2Q}{\omega_0C_1} \\
R_3 = \frac{2Q}{C_1\omega_0(4Q^2 + 1)}
\]

The gain \( K \) must be unity, but \( Q \) can be up to 10. Using these relations, compute the appropriate values of the resistors to achieve center frequency \( f_0 = 1 \text{ KHz} \), Gain \( K = 1 \) and \( Q = 10 \).

Choose the capacitors as \( C_1 = C_2 = 0.1 \mu\text{F} \) and \( C_3 = 2C_1 \). Plot \(|G(s)| \) versus frequency.

Solution using MATLAB is highly recommended.
4. The circuit of Figure 11.42 is a MFB second-order all-pass filter whose transfer function is

\[ G(s) = \frac{V_{out}(s)}{V_{in}(s)} = \frac{K(s^2 - a\omega_0 s + b\omega_0^2)}{s^2 + a\omega_0 s + b\omega_0^2} \]

where the gain \( K = constant \), \((0 < K < 1)\), and the phase is given by

\[ \phi(\omega) = -2\tan^{-1}\left(\frac{a\omega\omega_0}{b\omega_0 - \omega^2}\right) \]

The coefficients \( a \) and \( b \) can be found from

\[ \phi_0 = \phi(\omega_0) = -2\tan^{-1}\left(\frac{a}{b - 1}\right) \]

For arbitrary values of \( C_1 = C_2 \), we can compute the resistances from

\[ R_2 = \frac{2}{a\omega_0 C_1} \]
\[ R_1 = \frac{(1 - K)R_2}{4K} \]
\[ R_3 = \frac{R_2}{K} \]
\[ R_4 = \frac{R_2}{1 - K} \]

For \( 0 < \phi_0 < 180^\circ \), we compute the coefficient \( a \) from

\[ a = \frac{1 - K}{2K\tan(\phi_0/2)} \left[ -1 + \frac{4K}{1 - K} \cdot \tan^2(\phi_0/2) \right] \]
and for $-180^\circ < \phi_0 < 0^\circ$, from

$$a = \frac{1 - K}{2K \tan(\phi_0/2)} \left[ -1 - \sqrt{1 + \frac{4K}{I - K} \cdot \tan^2(\phi_0/2)} \right]$$

Using these relations, compute the appropriate values of the resistors to achieve a phase shift $\phi_0 = -90^\circ$ at $f_0 = 1$ KHz with $K = 0.75$.

Choose the capacitors as $C_1 = C_2 = 0.01$ $\mu$F and plot phase versus frequency.

Solution using MATLAB is highly recommended.

5. The Bessel filter of Figure 11.43 has the same configuration as the low-pass filter of Example 11.3, and achieves a relatively constant time delay over a range $0 < \omega < \omega_0$. The second-order transfer function of this filter is

$$G(s) = \frac{V_{out}(s)}{V_{in}(s)} = \frac{3K \omega_0^2}{s^2 + 3 \omega_0 s + 3 \omega_0^2}$$

![Figure 11.43. Circuit for Exercise 5](image)

Figure 11.43. Circuit for Exercise 5

where $K$ is the gain and the time delay $T_0$ at $\omega_0 = 2\pi f_0$ is given as

$$T_0 = T(\omega_0) = \frac{12}{13\omega_0} \text{ seconds}$$

We recognize the transfer function $|G(s)|$ above as that of a low-pass filter where $a = b = 3$ and the substitution of $\omega_0 = \omega_C$. Therefore, we can use a low-pass filter circuit such as that of Figure 11.43, to achieve a constant delay $T_0$ by specifying the resistor and capacitor values of the circuit.

The resistor values are computed from
Using these relations, compute the appropriate values of the resistors to achieve a time delay \( T_0 = 100 \, \mu s \) with \( K = 2 \). Use capacitors \( C_1 = 0.01 \, \mu F \) and \( C_2 = 0.002 \, \mu F \). Plot \( |G(s)| \) versus frequency.

Solution using MATLAB is highly recommended.

6. Derive the transfer function of a fourth-order Butterworth filter with \( \omega_c = 1 \, rad/s \).

7. Derive the amplitude-squared function for a third-order Type I Chebyshev low-pass filter with 1.5 \( dB \) pass band ripple and cutoff frequency \( \omega_c = 1 \, rad/s \).

8. Use MATLAB to derive the transfer function \( G(z) \) and plot \( |G(z)| \) versus \( \omega \) for a two-pole, Type I Chebyshev high-pass digital filter with sampling period \( T_S = 0.25 \, s \). The equivalent analog filter cutoff frequency is \( \omega_c = 4 \, rad/s \) and has 3 \( dB \) pass band ripple. Compute the coefficients of the numerator and denominator and plot \( |G(z)| \) with and without pre-warping.