Spanwise pressure coherence on prisms using wavelet transform and spectral proper orthogonal decomposition based tools

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1. Introduction

Gust response prediction for structures in turbulent flow has still fundamentally been based on the quasi-steady and strip theories. The former builds up the relationship between turbulence field and turbulence-induced forces, while the latter relates to the spatial distribution of turbulence-induced forces (or buffeting forces) on the structures. The spatial distribution of turbulence-induced forces can generally be described via either the correlation coefficient function in the time domain or the coherence function in the frequency domain. The spatial correlation and coherence of turbulence-induced forces is an essential issue in gust response theory and consequently affects the accuracy of structures' response prediction. Because the distribution of turbulence-induced forces is hard to measure in the wind tunnel, wind distribution has been used instead of force distribution for the sake of simplicity. It is also assumed that coherence of turbulence-induced forces is similar to that of wind turbulence. However, this assumption contains a lot of uncertainties because it does not account for the effects of wind-structure interaction and bluff body flow. Moreover, correspondence between turbulence coherence and force coherence in both the time and frequency domains has not yet been clarified. Recently, some physical measurements have indicated that force coherence is larger than turbulence coherence (e.g., Larose, 1996; Jakobsen, 1997; Kimura et al., 1997; Matsumoto et al., 2003). Higher coherence of turbulence-induced forces may cause underestimation of structures' gust response prediction. Moreover, practical formulae for force coherence have been based on Davenport's formula containing the parameters of spanwise separation and frequency (Jakobsen, 1997) or von Karman's formula adding the turbulence condition parameter (Kimura et al., 1997). Thus, it seems that the von Karman's formula is preferable to the Davenport's one, but it is complicated to apply and understand meaning of mathematical functions inside it. The mechanism of higher force coherence and effect of bluff body flow and temporal parameter still have not fully been clarified yet. Spanwise coherence of forces has generally been studied via a mean of surface pressure field because the buffeting forces can be estimated from the surface pressure by an integration operation, furthermore, the pressure field is directly measured and fundamentally related to an influence of the bluff body flow around models.

Several analytical tools have been developed for investigating the coherence of wind turbulence and pressure. The most common uses Fourier-transform-based coherence in which the correlations of surface fluctuating pressure and turbulence have been carried out on typical prisms with slenderness ratios of $B/D = 1$ and 5 in turbulent flow.
Fourier coherence is defined as the normalized correlation coefficient of two spectral quantities of $X(t)$ and $Y(t)$ in the frequency domain

$$\text{COH}_F^2(f) = \frac{\left< \text{Sy}(f) \right>^2}{\left< \text{Sy}^2(f) \right> \left< \text{Sx}^2(f) \right>},$$

(1)

where $\left< \cdot \right>$ is the absolute operator; $\left< \cdot \right>$ is the smoothing operator; $f$ is the Fourier frequency variable; $Sx(f)$, $Sy(f)$, $S_{xy}(f)$ are the Fourier auto-power spectra and Fourier cross power spectrum at/between two separated points, respectively, $Sx(f) = E[X(f)X^*(f)^{-1}]$; $Sy(f) = E[Y(f)Y^*(f)^{-1}]$; $S_{xy}(f) = E[X(f)Y^*(f)^{-1}]$; $E[]$ is the expectation operator; *t, T* are the complex conjugate and transpose operators; and $X(f)$, $Y(f)$ are the Fourier transform coefficients of time series $X(t)$, $Y(t)$. The Fourier coherence is normalized between 0 and 1 and is unity when two time series are fully correlated, and zero when two time series are uncorrelated in the frequency domain. Cross-correlations of pressure fields have been presented by some authors (e.g., Kareem, 1997; Larose, 2003), whereas coherence of pressure fields have been presented by these authors using Fourier transform-based tools. Fourier coherence is applicable for purely stationary time series, and no temporal information can be observed.

Recently, wavelet transform and its advanced tools have been applied to several topics in wind engineering. Wavelet transform has advantages for analyzing nonstationary events, especially for representing time series in the time–frequency plane. Corresponding to Fourier transform-based tools, high-order wavelet-transform-based ones have been developed, such as wavelet auto-power spectrum, wavelet cross power spectrum, wavelet coherence and wavelet phase difference. Wavelet transform coefficients have been applied to analyze time series of turbulence and pressure (e.g., Geurts et al., 1998), and wavelet-coherence-detected cross-correlation between turbulence and pressure (Kareem and Kijewski, 2002; Gurlay et al. 2003). In these studies, the traditional complex Morlet wavelet with fixed time–frequency resolution and no smoothing in time and scale have been used. Wavelet coherence has also been used to investigate effects of spanwise separations, frequency and intermittency on pressure coherence as well as comparison between turbulence and pressure coherence (Le et al., 2009). Using both Fourier coherence and wavelet coherence, Le et al. (2009) discussed pressure coherence based on the following points: (1) pressure coherence is higher than turbulence coherence due to the effect of bluff body flow on the model surface; (2) coherent structures of turbulence and pressure depend on parameters of turbulence condition, frequency, spanwise separation and bluff body flow; (3) pressure coherence is distributed intermittently in the time domain, and intermittency can be considered as a feature of pressure coherence; (4) high coherence events are distributed locally in the time–frequency plane and can be observed even at long separations and high frequencies, and the existence of localized high coherence events is also a feature of pressure coherence; (5) no simultaneous correspondence between high coherence events of turbulence and pressure has been observed in the time–frequency plane. However, intermittency and effect of time–frequency resolution on pressure coherence requires to be further investigated via wavelet coherence maps.

Proper orthogonal decomposition and its tools have been used in various applications in wind engineering. It has been developed into main branches in the time domain and the frequency domain (Solari and Carassale, 2000). Coherent structure of turbulence fields has been investigated using covariance-branched proper orthogonal decomposition (Lumley, 1970). Usage of the first covariance mode of the turbulence field can identify the coherent structure and hidden high-energy characteristics of the turbulence field. Furthermore, covariance modes and associated principal coordinates have also been used for studying cross correlation of turbulence and pressure (Tamura et al., 1997). However, spectral-branched proper orthogonal decomposition is promising for studying and standardizing pressure coherence thanks to orthogonal decomposition and low-order approximation of a coherence matrix of the pressure field in the frequency domain. In particular, independent spectral modes containing simultaneous frequency and space parameters can be used to investigate effects of bluff body flow on pressure coherence. Due to quietly different approach, there is no mathematical link between the wavelet transform and the proper orthogonal decomposition, furthermore, further investigation is required for feasibility of their mutual collaboration.

In this paper, pressure coherence has been investigated using new analytical tools based on the wavelet transform in the time–frequency plane and proper orthogonal decomposition in the frequency domain. Pressure coherences have been investigated via wavelet coherence and coherence mode for better understanding of the effects of intermittency, time–frequency resolution, wavelet function parameters and bluff body flow or chordwise pressure positions on pressure coherence. The modified complex Morlet wavelet with more flexibility in the time–frequency resolution analysis as well as the smoothing technique in both time and scale have been applied for wavelet coherence. Moreover, the coherence mode has been proposed from spectral-branched proper orthogonal decomposition. Surface pressures have been measured on some typical prisms with slenderness ratios $B/D=1$ and $5$ in turbulent flow.

## 2. Wavelet transform and wavelet coherence

### 2.1. Theoretical basis

The continuous wavelet transform of time series $X(t)$ is defined as the convolution operation between $X(t)$ and the wavelet function $\psi_{\tau,s}(t)$ (Daubechies, 1992)

$$W^\psi_X(\tau,s) = \int_{-\infty}^{\infty} X(t) \psi^*_{\tau,s}(t) dt$$

(2)

where $W^\psi_x(\tau,s)$ are the wavelet transform coefficients at translation $\tau$ and scale $s$ in the time–scale plane; $[,]$ denotes the convolution operator; $\psi_{\tau,s}(t)$ is the wavelet function at translation $\tau$ and scale $s$ of the basic wavelet function or mother wavelet $\psi(t)$, expressed as follows:

$$\psi_{\tau,s}(t) = \frac{1}{\sqrt{s}} \psi\left(\frac{t-\tau}{s}\right)$$

(3)

The wavelet transform coefficients $W^\psi_x(\tau,s)$ can be considered as a correlation coefficient and a measure of similarity between the wavelet function and the original time series in the time–scale plane. The wavelet scale has its meaning as an inverse of the Fourier frequency. Thus, the relationship between the wavelet scale and the Fourier frequency can be obtained

$$s = \frac{f_c}{f}$$

(4)

where $f_c$ is the wavelet central frequency. It is noted that Eq. (4) is satisfied at unit sampling frequency of the wavelet function. Thus, the sampling frequency of the time series and the wavelet function must be added in the relationship between the wavelet scale and the Fourier frequency.

### 2.2. Modified complex Morlet wavelet

The complex Morlet wavelet is the most commonly used for the continuous wavelet transform because it contains a harmonic component as analogous to the Fourier transform, which is better
adapted to capture oscillatory behavior in the time series. A modified form of the complex Morlet wavelet has been applied here for more flexible analysis of time–frequency resolution

\[
\psi(t) = \pi f_0^{-1/2} e^{i \pi f_0 t} e^{-t^2/\delta^2} \quad (5a)
\]

\[
\hat{\psi}(sf) = \pi f_0^{-1/2} e^{-\pi^2 s^2 f_0 f^2} \quad (5b)
\]

where \(\hat{\psi}(sf)\) is the Fourier transform coefficient of wavelet function and \(f_0\) is the bandwidth parameter. A fixed bandwidth parameter \(f_0=2\) is used in the traditional complex Morlet wavelet (Kareem and Kijewski, 2002; Gurley et al., 2003). Generally, the central frequency relates to the number of waveforms, whereas the bandwidth parameter relates to the width of the wavelet window.

2.3. Wavelet coherence

Corresponding to the Fourier-transform-based tools, one would like to develop wavelet transform-based tools such as wavelet auto-spectrum, wavelet cross spectrum, wavelet coherence of two time series \(X(t)\) and \(Y(t)\), based on their wavelet transform coefficients \(\hat{W}_X(s, i)\), \(\hat{W}_Y(s, i)\), which are defined by the following formulae:

\[
WPS_{XX}(s) = \langle \hat{W}_X(s) \hat{W}_X^*(s) \rangle \quad WPS_{YY}(s) = \langle \hat{W}_Y(s) \hat{W}_Y^*(s) \rangle \quad WPS_{XY}(s) = \langle \hat{W}_X(s) \hat{W}_Y^*(s) \rangle \quad WPS_{YY}(s) = \langle \hat{W}_Y(s) \hat{W}_Y^*(s) \rangle
\]

where \(WPS_{XX}(s)\), \(WPS_{YY}(s)\) are the wavelet auto-spectra of \(X(t)\) and \(Y(t)\); \(WPS_{XY}(s)\) is the wavelet cross spectrum between \(X(t)\) and \(Y(t)\); and \(\langle \ldots \rangle\) is the smoothing operator on both time and scale axes.

With respect to the Fourier coherence, the squared wavelet coherence of \(X(t)\) and \(Y(t)\) is defined as the absolute value squared of the smoothed wavelet cross spectrum, normalized by the smoothed wavelet auto-spectra (Torrence and Compo, 1998)

\[
WCO_{XY}(s) = \frac{\left| \langle \hat{W}_X(s) \hat{W}_Y^*(s) \rangle \right|^2}{\left( \langle \hat{W}_X(s) \hat{W}_X^*(s) \rangle \right)^2}
\]

(7)

where \(WCO_{XY}(s)\) is the wavelet coherence of \(X(t)\) and \(Y(t)\), and \(s^{-1}\) is used to normalize unit energy density.

Furthermore, wavelet phase difference is also computed from the wavelet cross spectrum

\[
\text{WPD}_{XY}(s) = \arctan \frac{\text{Im} \langle \hat{W}_X(s) \hat{W}_Y^*(s) \rangle}{\text{Re} \langle \hat{W}_X(s) \hat{W}_Y^*(s) \rangle}
\]

(8)

where \(\text{WPD}_{XY}(s)\) is the wavelet phase difference between \(X(t)\) and \(Y(t)\), and \(\text{Im}, \text{Re}\) are the imaginary and real parts of the wavelet cross spectrum of \(X(t)\) and \(Y(t)\).

2.4. Time–scale smoothing and end effect

Smoothing in both time and scale axes is inevitable for estimating wavelet spectra, wavelet coherence and wavelet phase difference. One would obtain more accuracy for the wavelet coherence by removing noise and conversion from local wavelet power spectrum to global wavelet power spectrum as well. A linear time-averaged wavelet power spectrum over a certain period at the time-shifted index \(i\) as well as the weighted scaled-averaged wavelet power spectrum over a scale range between \(s_1\) and \(s_2\) were proposed in Torrence and Compo (1998)

\[
\langle WPS^2_{XY}(s_j) \rangle = \frac{1}{n_i} \sum_{i=1}^{n_i} |WPS_{XY}(s_j)|^2
\]

(9a)

\[
\langle WPS^2_{YY}(s_j) \rangle = \delta j \delta t \sum_{i=1}^{n_j} |WPS_{YY}(s_j)|^2 / s_j
\]

(9b)

where \(i\) is the midpoint index between \(i_1\) and \(i_2\); \(n_i = i_2 - i_1 + 1\); \(j\) is the scaling index between \(j_1\) and \(j_2\); \(\delta j, \delta t\) are the empirical factors for scale averaging; and \(C_0\) is the empirical reconstruction factor.

Because the wavelet function applies finite window width on the time series, errors usually occur at two ends of the wavelet transform-based coefficient and spectrum, known as the end effect or signal padding. The influence of end effect is larger at low frequency and smaller at high frequency. The so-called cone of influence should be eliminated from the computed wavelet transform-based quantities. A simple solution is to wipe out the portions of results from the two ends of the wavelet transform coefficient and spectrum in the time axis. Estimated portions of the eliminated results at the two ends in the time domain can be referred in Kijewski and Kareem (2003).

2.5. Time–frequency resolution

The time–frequency resolution used in the wavelet transform is multi-resolution depending on frequency bands, in which high-frequency resolution and low time resolution are used for the low-frequency band, and inversely. The Heisenberg's uncertainty principle revealed that it is impossible to simultaneously obtain optimal time resolution and optimal frequency resolution. A narrow wavelet will have good time resolution but poor frequency resolution, while a broader wavelet has poor time resolution but good frequency resolution. The time–frequency resolution of the traditional Morlet wavelet has been discussed elsewhere (e.g., Kijewski and Kareem, 2003; Gurley et al., 2003). In the modified Morlet wavelet with additional bandwidth parameter \(f_0\), the time–frequency resolution can be extended as follows:

\[
\Delta f = \frac{\Delta f_0}{s} = \frac{f}{2\pi f_0 \sqrt{s_0}}
\]

(10a)

\[
\Delta t = \frac{s\Delta f_0}{2f}
\]

(10b)

where \(\Delta f_0, \Delta f_0\) are the frequency resolution and time resolution of the modified Morlet wavelet, and \(f\) is the analyzing frequency. The optimum relationship between frequency and time resolution \(\Delta f_0\Delta f_0 = 1/4\pi\) is considered.

One can adjust the wavelet central frequency \(f_c\) and the bandwidth parameter \(f_0\) to obtain the desired frequency resolution and the desired time resolution at the analyzing frequency.

3. Proper orthogonal decomposition and coherence modes

3.1. Theoretical basis

Proper orthogonal decomposition is considered as the optimum approximation of zero-mean multi-variate random fields via basic orthogonal vectors and uncorrelated random processes (principal coordinates). In this manner, fluctuating pressure field \(p(t)\) represented as \(N\)-variate random pressure process
where \( a_i(t) \) is the \( i \)-th principal coordinate as zero-mean uncorrelated random process; \( \varphi_i \) is the \( i \)-th basic orthogonal vector \( a_i(t) = [a_1(t), a_2(t), \ldots, a_N(t)] \); and \( N \) truncated number of low-order modes \( (N \ll N) \).

Mathematical expression of optimality of multi-variate random fields can be expanded in the form of equality (Lumley, 1970)

\[
\int_R \rho_k(\tau) \Phi d\tau = \lambda \Phi
\]

where \( \rho_k(\tau) = [\rho_k(p_i, p_j, \tau)] \) is the covariance matrix; \( \rho_k(p_i, p_j, \tau) \) is the covariance value between two pressure points \( p_i, p_j \); \( \tau \) is the time lag; \( \lambda \) is the weighted coefficient; and \( \nu \) is the variable space.

Solution of the orthogonal space function \( \Phi \) can be determined via the eigen problem

\[
\rho_k(0) \Phi = \Lambda \Phi
\]

where \( \rho_k(0) \) is the zero-time-lag covariance matrix of random field defined as \( \rho_k(0) = [\rho_k(0)_{p_i, p_j}, \rho_k(0)_{p_i, p_j} = E\rho_k(p_i, p-j)] \); \( \Lambda \) is the diagonal covariance eigenvalue matrix \( \Lambda = \text{diag}(\lambda_1, \lambda_2, \ldots, \lambda_N) \); and \( \Phi \) is the covariance eigenvector matrix containing independent covariance modes \( \varphi_i \). Expressions in Eqs. (11) and (13), are known as covariance-branched proper orthogonal decomposition in the time domain.

### 3.2. Spectral proper orthogonal decomposition and coherence mode

Spectral proper orthogonal decomposition has been used to approximate characterized matrices of random fields in the frequency domain. Usually, a squared cross spectral matrix of fluctuating pressure fields is built and then the spectral space function \( \Phi(f,j) \) (as function of space and frequency) can be determined basing on the eigen problem of the cross spectral matrix \( S_p(f,j) \)

\[
S_p(f,j) \Phi(f,j) = \Lambda(f) \Phi(f,j)
\]

where \( \Lambda(f) \) is the spectral eigenvalue matrix \( \Lambda(f) = \text{diag}(\lambda_1(f), \lambda_2(f), \ldots, \lambda_N(f)) \); and \( \Phi(f,j) \) is the spectral space function or spectral eigenvector matrix \( \Phi(f,j) = [\varphi_1(f,j), \varphi_2(f,j), \ldots, \varphi_N(f,j)] \), known as spectral modes.

Spectral proper orthogonal decomposition is extended to treat a coherence matrix of fluctuating pressure field, which is defined as \( C_p(f,j) = [C_{OH}(f,j)] \) where \( C_{OH}(f,j) \) is the coherence function between two fluctuating pressure points \( p_i, p_j \). It is noted that the coherence matrix is a rectangular frequency-dependent positive-definite matrix containing space information in both chordwise and spanwise directions. Singular value decomposition can be used to orthogonally decompose the rectangular coherence matrix, in which the spectral space functions \( \Phi(f,j) \) are computed

\[
C_p(f,j) = \Lambda(f) \Phi(f,j)
\]

where \( \Lambda(f) \) is the diagonally singular value matrix, and \( \Phi(f,j) \) is the singular vector matrices \( \Phi(f,j) = [\varphi_1(f,j), \varphi_2(f,j), \ldots, \varphi_N(f,j)] \); \( \Gamma(f,j) = [\gamma_1(f,j), \gamma_2(f,j), \ldots, \gamma_N(f,j)] \), so-called coherence modes containing space variable \( f \) and frequency variable \( j \).

The spanwise coherence matrix of the fluctuating pressure field can be approximated using a limited number of low-order coherence modes

\[
C_p(f,j) \approx \sum_{i=1}^{N} \psi_i(f,j) \Delta f \varphi_i(f,j)^T, \quad \Delta f < N.
\]

Significantly, independent low-order coherence modes can represent the spanwise pressure coherence of the fluctuating pressure fields in both chordwise and spanwise spaces, as well as frequency. The importance of the coherence modes can be evaluated using the so-called energy contribution. The energy contribution of the \( i \)-th coherence mode on the total energy of the pressure field can be determined as a proportion of spectral eigenvalues on cut-off frequency range as

\[
E_{\psi_i(f,j)} = \sum_{k=0}^{f_{cut}} \lambda_k(f_k) / \sum_{i=1}^{N} \sum_{k=0}^{f_{cut}} \lambda_k(f_k)
\]

where \( E_{\psi_i(f,j)} \) is the energy contribution of \( i \)-th coherence mode; \( \lambda_i \) is the \( i \)-th spectral eigenvalue; and \( f_{cut} \) is the cut-off frequency. Because singular value decomposition is fast decoding, thus the first coherence modes usually contain dominant energy and they can be used to investigate the pressure coherence.

### 4. Surface pressure measurements on prisms

Physical measurements of ongoing turbulence and surface pressure were carried out on several prisms with typical slender-ness ratios of \( B/D = 1 \) and \( 5 \) (\( B, D \) is the width and depth of prisms). Isotropic turbulence flow was generated artificially using grid devices installed upstream of the prisms. The turbulence intensities of two turbulence components were \( I_x = 11.56\%, I_y = 11.23\% \). Pressure taps were arranged on one surface of the prisms, 10 on prism \( B/D = 1 \) and 19 on prism \( B/D = 5 \) in the chordwise direction, and with separations \( y = 25, 75, 125 \) and 225 mm from a reference pressure line at \( y = 0 \) mm in the spanwise direction (see Fig. 1).

Both longitudinal (\( u \)) and vertical (\( w \)) turbulence components of the fundamental turbulence flow (without prisms) were measured by a hot-wire anemometer using \( x \)-type probes, while fluctuating surface pressures were measured on the prisms by a multi-channel pressure measurement system. Both turbulence components and pressures were simultaneously obtained in order to investigate their compatibility in the time–frequency plane. Electric signals were passed through 100 Hz low-pass filters, then \( A/D \) converted at a sampling frequency at 1000 Hz at 100-s intervals.

### 5. Results and discussion

Bluff body flow is generally defined as flow around a bluff body’s surface due to interaction between fundamentally ongoing turbulence and the bluff body, including not only chordwise flow behaviors at leading edge, trailing edge, on surface and at wake of the bluff body such as formatting separated and reattached flows, separation bubble and vortex shedding, but also convective flow in the spanwise direction. It is generally agreed that prism \( B/D = 1 \) is favorable for formation of Karman vortex shedding in the wake, while prism \( B/D = 5 \) is typical for formatting separated and reattached flows on the surface and a separation bubble in the leading edge region as well (e.g., Okajima, 1990; Bruno et al., 2010). Fourier-transform-based coherence of turbulence and coherences of pressures on rectangular prisms and girders have been investigated by many authors (e.g., Larose, 1996; Jakobsen, 1997; Kimura et al., 1997; Matsumoto et al., 2003; Le et al., 2008). They showed that pressure coherence decreases with increase in spanwise separation and frequency, and that pressure coherence is larger than turbulence coherence for the same separation and frequency. They also argued for significant
influences of bluff body flow and the ongoing turbulence condition on pressure coherence (Le et al., 2009). Pressure coherence seems to be larger at higher turbulence intensities, and is also larger in the trailing edge region of prism $B/D=5$. This assumes that secondary convective flow might be enhanced in the separation bubble region of prism $B/D=5$ and consequently increases pressure coherence.

![Fig. 1. Experimental models and pressure tape layout.](image)

![Fig. 2. Wavelet coherence maps of pressure and turbulence at various spanwise separations: (a) $B/D=1$, (b) $B/D=5$ and (c) w-turbulence.](image)
Temporal–spectral pressure coherence of fluctuating pressure fields on prisms $B/D = 1$ and 5 has been investigated using wavelet coherence. The wavelet transform coefficients, the wavelet auto-spectra and the wavelet cross spectra of the pressure have been computed from Eq. (6) before wavelet coherence in Eq. (7) was estimated. Time–frequency smoothing as in Eq. (9) and end-effect elimination were carried out to estimate the wavelet coherences of turbulence and pressure. Fig. 2 shows the wavelet coherences of both w-turbulence and pressure on prisms $B/D = 1$ and 5 at spanwise separations $y = 25, 75$ and 125 mm, in the 1–50 Hz frequency band and 5–95 s intervals. Here, 5-s intervals at two ends of the computed wavelet coherence are eliminated for treatment of the end effect. Obviously, the wavelet coherence maps provide information of pressure coherence in both the time and frequency domains, whereas only information in the frequency domain can be observed in Fourier coherence. Some following discussions are given from the results of Fig. 2. Firstly, like previous results based on Fourier coherence (e.g., Matsumoto et al., 2003; Le et al., 2009), the wavelet coherence maps via color indicator indicate that the coherences of turbulence and pressure reduces with increase in spanwise separation and frequency, and pressure coherence is larger than turbulence coherence at the same separations and the same frequencies. Secondly, pressure coherence and turbulence coherence are distributed locally and intermittently in the time–frequency plane. This implies that intermittency is a characteristic of both turbulence coherence and pressure coherence in the time–frequency plane. Thirdly, high coherence events are still observed in both turbulence and pressure coherences even at distant separations and in high-frequency bands, but localized in small time–frequency areas. Intermittency and localized high coherence events of turbulence coherence and pressure coherence can be clarified in wavelet coherence maps, but not observed from conventional Fourier coherence and empirical formulae. Finally, no correspondence in the time–frequency plane between high coherence events of pressure coherence and of turbulence coherence can be clarified, although pressure and the turbulence were simultaneously measured.

Fig. 3 shows a more detailed wavelet coherence map of pressures on prism $B/D = 1$ at spanwise separation $y = 25$ mm with new concepts. So-called globally averaged wavelet coherence in the frequency domain is defined as the average of all local wavelet coherences over an entire time domain (here the time interval is 5–95 s). Moreover, the so-called wavelet coherence ridge in the time domain is defined as dominant wavelet coherence at a certain frequency, which is searched from a peak of the globally averaged wavelet coherence in the frequency domain (as shown by the dotted line in Fig. 3). The averaged wavelet coherence represents global frequency-dependent information of the wavelet coherence map in the frequency domain, which can be compared with the Fourier coherence. The wavelet coherence ridge represents localized information of the wavelet coherence map in the time domain, in which time-dependent characteristics and intermittency of the wavelet coherence can be observed. For instance the wavelet coherence ridge of the pressures on prism $B/D = 1$ at separation $y = 25$ mm indicates a local discontinuity and low coherence events of pressures at time points 11, 57 and 87 s.

Fig. 4 shows the globally averaged wavelet coherence and the wavelet coherence ridges of the fluctuating pressure fields on prism $B/D = 1$ at different spanwise separations $y = 25, 75, 125$ and 225 mm. Obviously, the average wavelet coherence of pressures decreases with increase in spanwise separation (see Fig. 4a). However, an overestimation of averaged wavelet coherence is observed in the high-frequency band, which might be caused by low-frequency resolution in the high-frequency band and averaging in the time domain. Intermittency and local low-coherence events of pressure coherence in the time domain seem to increase with increase in spanwise separation. Moreover, very low coherence can be observed locally in the wavelet coherence ridges at

![Image](225x150 to 441x339)

Fig. 3. Averaged wavelet coherence and wavelet coherence ridge ($B/D = 1, y = 25$ mm).

higher separations (see Fig. 4b). It is also observed that there is correspondence between low coherence events of pressure coherences in the time points at the spanwise separations.

Fig. 4. Averaged wavelet coherences and wavelet coherence ridges of pressures at various spanwise separations ($B/D = 1$): (a) averaged wavelet coherence at various spanwise separations and (b) wavelet coherence ridges at various spanwise separations.

Fig. 5 compares the globally averaged wavelet coherence and the Fourier coherence of pressure and turbulence. There is agreement between them at in the low-frequency band, but difference in the higher-frequency band. As in previous studies using Fourier coherence, the wavelet coherence of pressure is also larger than that of turbulence. Because the averaged wavelet coherence is smoother than Fourier coherence, it seems to be more appropriate for fitting and estimating parameters of empirical coherence equations.

The effect of time–frequency resolution on wavelet coherence of pressure has been investigated by changing the central frequency $f_c$ and the bandwidth parameter $f_b$ in the modified wavelet transform.

Table 1

<table>
<thead>
<tr>
<th>Resolution parameters and Frequency</th>
<th>5 Hz</th>
<th>10 Hz</th>
<th>20 Hz</th>
<th>30 Hz</th>
</tr>
</thead>
<tbody>
<tr>
<td>$f_b = 2, f_c = 1$</td>
<td>1.25</td>
<td>0.14</td>
<td>2.50</td>
<td>0.07</td>
</tr>
<tr>
<td>$f_b = 2, f_c = 5$</td>
<td>0.25</td>
<td>0.71</td>
<td>0.50</td>
<td>0.35</td>
</tr>
<tr>
<td>$f_b = 5, f_c = 1$</td>
<td>0.13</td>
<td>1.41</td>
<td>0.25</td>
<td>0.71</td>
</tr>
<tr>
<td>$f_b = 5, f_c = 5$</td>
<td>0.79</td>
<td>0.22</td>
<td>1.58</td>
<td>0.11</td>
</tr>
<tr>
<td>$f_b = 5, f_c = 10$</td>
<td>0.16</td>
<td>1.12</td>
<td>0.32</td>
<td>0.56</td>
</tr>
</tbody>
</table>

Fig. 5 compares the globally averaged wavelet coherence and the Fourier coherence of pressure and turbulence. There is agreement between them at in the low-frequency band, but difference in the higher-frequency band. As in previous studies using Fourier coherence, the wavelet coherence of pressure is also larger than that of turbulence. Because the averaged wavelet coherence is smoother than Fourier coherence, it seems to be more appropriate for fitting and estimating parameters of empirical coherence equations.

The effect of time–frequency resolution on wavelet coherence of pressure has been investigated by changing the central frequency $f_c$ and the bandwidth parameter $f_b$ in the modified wavelet transform.

Fig. 6. Averaged wavelet coherence and wavelet coherence ridges at various time–frequency resolutions ($B/D = 1$, $y = 25$ mm): (a) averaged wavelet coherence at various time–frequency resolutions and (b) wavelet coherence ridges at various time–frequency resolutions.

Fig. 5 compares the globally averaged wavelet coherence and the Fourier coherence of pressure and turbulence. There is agreement between them at in the low-frequency band, but difference in the higher-frequency band. As in previous studies using Fourier coherence, the wavelet coherence of pressure is also larger than that of turbulence. Because the averaged wavelet coherence is smoother than Fourier coherence, it seems to be more appropriate for fitting and estimating parameters of empirical coherence equations.

The effect of time–frequency resolution on wavelet coherence of pressure has been investigated by changing the central frequency $f_c$ and the bandwidth parameter $f_b$ in the modified wavelet transform.

complex Morlet wavelet. It is noted that the time–frequency resolution of the wavelet function and the time series changes with analyzed frequency bands at fixed parameters of the wavelet function used in the continuous wavelet transform. The frequency resolution decreases with increase in analyzed frequency band, whereas the time resolution increases with increase in analyzed frequency band. Good frequency resolution accompanies poor time resolution, and inversely. But one would like to apply lower-frequency resolution and a wider window in the low-frequency band, and higher-frequency resolution and a narrower window in the high-frequency band. The time–frequency resolution computed at some analyzed frequencies with several pairs of central frequency and bandwidth parameter is given in Table 1 after Eq. (10). This indicates that the time–frequency resolution changes with the analyzing frequency. Furthermore, the frequency resolution decreases with increase in analyzed frequency. Averaged wavelet coherence and wavelet coherence ridges at investigated time–frequency resolutions with respect to prism \( B/D = 1 \) and spanwise separation \( y = 25 \text{ mm} \) are shown in Fig. 6. It is observed that the parameters of the wavelet function and the time–frequency resolution greatly influence the averaged wavelet coherence in the frequency domain and the wavelet coherence ridges in the time domain as well. Of the two parameters in the modified complex Morlet wavelet, moreover, the wavelet central frequency has a stronger influence on the wavelet coherence. Lower center frequency produces higher wavelet coherence, while a higher bandwidth parameter seems to produce higher wavelet coherence (see Fig. 6a). Intermittency of the wavelet coherence ridges in the time domain has been investigated with the time–frequency resolution and the parameters of the wavelet function as shown in Fig. 6b. More intermittency and low wavelet coherence are observed at high central frequency. It seems that high and low wavelet coherence events with the same central frequencies appear at similar time points in the time domain.

Effects of bluff body flow on the prisms’ surfaces or chordwise pressure positions on the pressure coherence of the fluctuating pressure fields on the prisms has been considered via globally averaged wavelet coherences. Fig. 7 shows the averaged wavelet coherence at chordwise pressure positions 3, 5, 7, 9 on prism \( B/D = 1 \) and positions 3, 7, 11, 15 on prism \( B/D = 5 \), at spanwise separations \( y = 25 \text{ mm} \). It is observed that the wavelet coherences at the investigated chordwise pressure positions on prism \( B/D = 1 \) seem to differ only in the very low-frequency band, while significant differences in wavelet coherences at the chordwise pressure positions are observed on prism \( B/D = 5 \). Specifically, wavelet coherence decreases in the low-frequency band, but stays uniform outside it when the chordwise pressure positions move from the leading edge to the trailing edge in prism \( B/D = 1 \) (see Fig. 7a). This can be explained by the uniform bluff body flow over the entire surface of prism \( B/D = 1 \). In prism \( B/D = 5 \), strong and dominant wavelet coherence is observed at chordwise position No. 3 inside the separation bubble region; a complicated

![Fig. 7. Averaged wavelet coherence at chordwise pressure positions (B/D = 1, B/D = 5, y = 25 mm): (a) effect of chordwise pressure positions or bluff body flow on B/D = 1 and (b) effect of chordwise pressure positions or bluff body flow on B/D = 5.](image)

![Fig. 8. First five singular values: (a) B/D = 1 and (b) B/D = 5.](image)
change is observed at pressure position 7 near the reattachment region of the bluff body flow; and a sudden reduction is observed at pressure positions 11 and 15 after the reattachment region and near the trailing edge region (see Fig. 7b). It is assumed that the wavelet coherence is relatively dominant at the separation bubble positions, and relatively small at the reattachment region positions and the trailing edge positions. An influence of the pressure positions in the chordwise direction on the spanwise pressure coherence is apparently observed. Thus, effect of the bluff body flow on spanwise pressure coherence can be reasoned for higher mechanism of the pressure coherence over the turbulence coherence.

Spatial–spectral coherence modes have been computed from singular value decomposition in Eq. (15) of the coherence matrices of fluctuating pressure fields on prisms $B/D=1$ and 5 in turbulent flow. It is noted that two types of spatial–spectral coherence, spanwise coherence and chordwise coherence, are extracted from two spectral space functions in chordwise and spanwise directions. Singular values also obtained from singular value decomposition of the coherence matrices are used to evaluate the energy contribution of the coherence modes as given in Eq. (16), especially the energy contribution of the first coherence modes. The energy contributions of the first coherence modes (both the first spanwise coherence mode and the first chordwise coherence mode) of prisms $B/D=1$ and 5 have been estimated as 56% and 50% with respect to a cut-off frequency of 100 Hz. If the narrowed range 0–10 Hz is taken, the first coherence modes of prisms $B/D=1$ and 5 contribute up to 89% and 73% of the total energy of the fluctuating pressure fields. The first coherence modes are meaningful for investigating characteristics of pressure coherence due to their orthogonality and dominant energy contribution.

Coherence matrices of the fluctuating pressure fields on prisms have been constructed before the spectral proper orthogonal decomposition has been applied to determine the singular values and the coherence modes. Fig. 8 shows the first five singular values of the fluctuating pressure fields for the prisms $B/D=1$ and 5 in frequency band 0–50 Hz. Energy contribution of the first coherence modes of the prisms has been estimated following the Eq. (17), respectively, 56% and 50% in the computed frequency range. If a low-frequency range 0–10 Hz is taken into account, the first coherence modes of the prisms $B/D=1$ and 5 hold up to 89% and 73% of the total energy of the pressure fields. Their dominant energy contribution proves that the first coherence modes could be used to represent characteristics of the spanwise coherence of the fluctuating pressure fields on prisms.

Fig. 9 shows the first spanwise and chordwise coherence modes of the fluctuating pressure fields of prisms $B/D=1$ and 5 with respect to the effect of spanwise separation and of chordwise pressure position. All the chordwise pressure positions and the spanwise separations $y=25, 50, 75, 100, 125, 150, 175, 200$ and 225 mm have been taken to compute the coherence modes. It is also observed from the first spanwise coherence mode that the pressure coherence decreases with increase in spanwise separation and observed frequency, while the first chordwise coherence mode indicates the influence of chordwise position and bluff body flow. Local high coherence can be observed in the leading edge region and the separation bubble region of prism $B/D=5$, whereas the coherence seems to be more uniformly

![Fig. 9. First coherence modes of pressure: (a) first spanwise coherence mode and effect of spanwise separation and (b) first chordwise coherence mode and effect of chordwise pressure position.](image)

distributed over all chordwise positions of prism $B/D=1$. This implies that secondary convective flow enhanced at the separation bubble region of prism $B/D=5$ might be a cause for this local high pressure coherence. Because the coherence modes contain spatial–spectral information of spanwise separations, observed frequencies and chordwise positions, they can be used to map intrinsic characteristics of pressure coherence.

6. Conclusion

Spanwise pressure coherence of fluctuating pressure fields on typical prisms $B/D=1$ and 5 has been investigated using wavelet coherence and coherence modes, by which the pressure coherence has been mapped in the time–frequency plane and the space–frequency plane. It is shown that not only spanwise separation and frequency influence pressure coherence, but also bluff body flow on the surface of the prisms. This has been observed via the coherence mode, and it shows that enhanced convective flow in the separation bubble region on prism $B/D=5$ causes local high-pressure coherence in this region. Moreover, the effects of bluff body flow and convective flow are reasons for the higher coherence mechanism of pressure coherence over turbulence coherence. Intermittency in the time domain and localized high coherence events of pressure coherence have been observed in wavelet coherence maps in the time–frequency plane, globally averaged wavelet coherence in the frequency domain and a wavelet coherence ridge in the time domain. It is indicated that the intermittency and localized high coherence are intrinsic characteristics of pressure coherence. Time–frequency resolution of the analyzed wavelet function significantly affects wavelet coherence and its temporal–spectral distribution in the time and frequency domains. Thus, analysis of time–frequency resolution should be carefully considered for computing wavelet coherence. Smoothing in both time and scale is also required for accuracy of wavelet coherence. Furthermore, use of the modified complex Morlet wavelet is preferable due to its adaptability and flexibility in analysis of time–frequency resolution.

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