Modal parameter estimation from ambient data using time-frequency analysis

T.H. Le¹, Y. Tamura², A. Yoshida³
¹Tokyo Polytechnic University, Wind Engineering Research Center
1583 Iiyama, Atsugi, Kanagawa 243-0297, Japan
email: thle@arch.t-kougei.ac.jp

²Tokyo Polytechnic University, Department of Architectural Engineering
1583 Iiyama, Atsugi, Kanagawa 243-0297, Japan
email: yukio@arch.t-kougei.ac.jp

³Tokyo Polytechnic University, Department of Architectural Engineering
1583 Iiyama, Atsugi, Kanagawa 243-0297, Japan
email: yoshida@arch.t-kougei.ac.jp

Abstract
Modal parameter estimation of randomly and ambient excited structures using output-only system identification techniques has become a recently essential issue for assessment of engineering structures, structural health monitoring and structural control. A number of mathematical models on the output-only identification techniques have been developed in the time domain, in the frequency domain and in the most recent time-frequency domains. Time-frequency analysis using wavelet transform advantages to represent measured response in the simultaneous time and frequency domains which facilitate for estimating both natural frequencies in the frequency domain and damping in the time domain. However, the output-only identification using the time-frequency analysis have difficulties such as time-frequency resolution analysis for damping estimation and close frequencies, especially for extracting high-order and low-energy modal parameters and so on. This paper presents theoretical bases of the wavelet transform for output-only system identification of the ambient data with emphasis on extracting the natural frequencies and damping. Full-scale measurements have been carried on a five-storey steel structure. Some refinement techniques using discrete wavelet decomposition with multi-resolution analysis, empirical mode decomposition as well as using broadband and narrowband filtering are also proposed for estimating the modal parameters of high-order mode and close-frequency problems. Modified complex Morlet wavelet, furthermore, is used for flexible adaptation on the time and frequency resolutions.

1 Introduction
Modal parameter estimation of randomly and ambient excited structures using the output-only identification methods is essential for structural health monitoring and structural control of engineering structures. So far, a number of mathematical models on the output-only identification methods have been developed and mainly classified by either parametric methods in the time domain or nonparametric ones in the frequency domain. Parametric methods in the time domain such as Ibrahim time domain, eigensystem realization algorithm, random decrement technique, and the most preferable stochastic subspace identification and the nonparametric methods in the frequency domain such as peak picking or basic frequency method, frequency domain decomposition and enhance frequency domain decomposition have been popularly used for the output-only system identification. Generally, each identification method in either the time domain or the frequency domain has its own advantage and limitation. Concretely, the
frequency domain-based methods are advantageous to extract natural frequencies and mode shapes but troublesome estimation for damping, whereas the time-domain-based methods require selection of parameters and even prior information on system’s natural frequencies. Recently, new approaches in the time-frequency analysis based on wavelet transform [2, 6, 7, 9–11, 13] and Hilbert-Huang transform [4, 7] have been developed for the output-only identification methods in the simultaneous time-frequency domains. Preferable advantage of the time-frequency analysis is to facilitate for estimating the natural frequencies in the frequency domain and the damping ratios in the time domain. Furthermore, the time-frequency analysis-based modal estimation methods have their solely powerful capacity for treatment of non-stationary, transient and non-linear inputs and outputs.

Wavelet transforms has recently developed basing on a convolution operation between a time series signal and a basic wavelet function which allows the signal to be represented in the time-scale (frequency) domains, also known as the time-frequency analysis [1]. The wavelet transform advantages to conventional Fourier transform and its modified version as short-time Fourier transform in analyzing non-stationary, non-linear and intermittent signals with simultaneous time-frequency information and flexible multi-resolution concept. The wavelet transform has been classified into two quite different branches of either the continuous wavelet transform or the discrete wavelet transform depending on discretized manner of the time-frequency domains and characteristic of used basic wavelets. Continuous wavelet transform has been applied for the modal parameter estimation and the output-only identification from the randomly excited structures (ex., [2, 5, 9, 10]). Use of the discrete wavelet transform for the modal parameter estimation is rare and inadequate due to its different approach, but one applied the multi-resolution analysis of the discrete wavelet transform to propose a discrete wavelet-logarithmic decrement formula to estimate the damping [6]. However, the wavelet transform still exists some main difficulties for the modal parameter estimation of the real structures as follows: (1) Analyzing time-frequency resolutions; (2) Extracting close frequencies; (3) Estimating modal parameters of the high-order modes; (4) Applying to practical data of real structures with full sources of noises and influence of excitation; and (5) Influencing on damping estimation from selections of time duration and of computed time resolution. The wavelet transform easily detects dominant and high-energy spectral components in the response signals, in the other word, it is convenient to extract modal parameters of few low-order modes containing high spectral energies. But, there are many structural modes existing in practical structures, the modal parameters of high-order modes in high frequency band are hardly detected via the wavelet transform if no further supplemental treatment on the frequency resolution is applied. In these cases, refinement techniques for the flexible frequency resolution at certain analyzed bandwidths should be required to detect and extract the high-order modal parameters. Furthermore, the traditional complex Morlet wavelet with a parameter of central frequency only is mostly used for the continuous wavelet transform, however, this traditional wavelet is not facilitated to cope with high frequency resolutions in high frequency bands, close frequency identification problem, and high-mode parameters where sophisticated and careful analysis of the time-frequency resolutions must be required. Modified complex Morlet wavelet has been discussed by some authors (e.g. [5, 13]) to give comprehensive approach for analysis of the time-frequency resolutions.

The paper presents theoretical bases of the wavelet transform-based method for the modal parameter estimation and the output-only system identification of the randomly and ambient excited structures with emphasis on analysis of time-frequency resolutions for of high-order modes as well as on influence of time-frequency resolutions on the accuracy of damping estimation. Full-scale measurements have carried on a five-storey steel building. Modified complex Morlet wavelet is used for the continuous wavelet transform, while Daubechies wavelet D5 exploited for the discrete wavelet transform. Refinement techniques have been proposed in order to analyze the frequency resolution and to determine the modal parameters of the high-order modes.
2 Continuous wavelet transform

2.1 Definition

The continuous wavelet transform of given signal $x(t)$ is defined as the convolution operation between signal $x(t)$ and wavelet function $\psi_{\tau,s}(t)$ [1]:

$$W_\psi^x(\tau, s) = \int_{-\infty}^{\infty} x(t) \psi^*_\tau,s(t) dt$$

(1)

where $W_\psi^x(s, \tau)$: wavelet transform coefficients at translation $\tau$ and scale $s$ in the time-scale plane; asterisk * means complex conjugate; $\psi_{\tau,s}(t)$: wavelet function at translation $\tau$ and scale $s$ of basic wavelet function $\psi(t)$, or mother wavelet:

$$\psi_{\tau,s}(t) = \frac{1}{\sqrt{s}} \psi\left(\frac{t - \tau}{s}\right)$$

(2)

The basic wavelet function or mother wavelet satisfy such following conditions as oscillatory function with fast decay toward zero, zero mean value, normalization and admissibility condition as follows:

$$\int_{-\infty}^{\infty} \psi(t) dt = 0; \quad \int_{-\infty}^{\infty} |\psi(t)|^2 dt = 1;$$

(3a)

$$C_\psi = \int_{-\infty}^{\infty} |\hat{\psi}(\omega)|^2 d\omega < \infty$$

(3b)

The wavelet transform coefficients can be considered as a correlation coefficient and a measure of similitude between the wavelet and the signal in the time-scale plane. The higher coefficient is, the more the similarity. It is noted that the wavelet scale is not a Fourier frequency, but revealed as an inverse of frequency. Accordingly, a relationship between the Fourier frequency and wavelet scale can be approximated:

$$f = \frac{f_c}{s}$$

(4)

where $f$: Fourier frequency; $s$: wavelet scale; and $f_c$: central frequency.

2.2 From traditional to modified complex Morlet wavelets

Traditional either real or complex Morlet wavelet is the most commonly used for the continuous wavelet transform so far, but the complex Morlet wavelet is preferable due to its containing of harmonic component and analog to the Fourier transform, in which the traditional complex Morlet wavelet and its Fourier transform are given as follows [2, 7, 9–11]:

$$\psi(t) = (2\pi)^{-1/2} \exp\left(2\pi if_c t\right) \exp\left(-t^2/2\right)$$

$$\hat{\psi}(sf) = (2\pi)^{-1/2} \exp\left(2\pi^2 (sf - f_c)^2\right)$$

(5a)

where $\psi(t), \hat{\psi}(sf)$: complex Morlet wavelet and its Fourier transform coefficient.

It is noted that only the central frequency regulates to the time and frequency resolutions in the traditional complex Morlet wavelet. In order to be flexibly adjustable on the time-frequency resolution, a modified complex Morlet wavelet has been used by [5, 13] as follows:
\[ \psi(t) = (\pi f_b)^{-0.5} \exp(j2\pi f_c t) \exp(-t^2 / f_b) \]  
(6a)

\[ \psi(sf) = \exp(-\pi^2 f_b (sf - f_c)^2) \]  
(6b)

where \( f_b \): bandwidth parameter. As a result, an analysis of time and frequency resolutions is facilitated by using both the central frequency and the bandwidth parameter.

**Figure 1: Modified Morlet wavelet with some parameters**

Investigations on the modified Morlet wavelet with some certain parameters (\( f_c \) and \( f_b \)) are shown in Figure 1.

### 2.3 Time-frequency resolution, smoothing and signal padding

Time-frequency resolution used in the wavelet transform is multi-resolution depending on frequency bands, in which high frequency resolution and low time resolution are used for low frequency band, and inversely. The Heisenberg’s uncertainty principle revealed us that it is impossible to simultaneously obtain optimal time resolution and optimal frequency resolution. Analysis of the time-frequency resolution is inevitable for the close frequency identification and the high-mode parameter identification using the wavelet transform. The time-frequency resolution of the traditional Morlet wavelet was discussed in somewhere (e.g. [2]). The time and frequency resolutions of the modified Morlet wavelet can be expressed as follows [13]:

\[ \Delta f_\psi = \frac{1}{2\pi \sqrt{f_b}} \]  
(7a)

\[ \Delta t_\psi = \frac{\sqrt{f_b}}{2} \]  
(7b)

where \( \Delta f_\psi, \Delta t_\psi \): frequency resolution and time resolution of the modified Morlet wavelet. Here, relationship between the frequency and time resolutions is \( \Delta f_\psi \Delta t_\psi = 1/4\pi \) considered as optimal product, normally we have the relationship \( \Delta f_\psi \Delta t_\psi \geq 1/4\pi \).

In the interrelation between the Fourier frequency and the wavelet central frequency, wavelet scale as shown in the Eq.(4), we have \( s = f_c / f \), one obtain the resolutions of time and frequency:
The wavelet transform coefficient is also determined through this smoothing. Either linear or logarithmic smoothing technique in the time domain and the scale domain can be exploited, which can be referred in [2, 12]:

Because the wavelet transform coefficient deals with finite-length signals, errors and bias values usually occur at two ends of signals, known as the end effect or signal padding. This signal padding must be eliminated from the computed wavelet transform. One simple solution to eliminate the end effect is to truncate number of results at two ends of the signal after the wavelet coefficient is computed. Removed number, however, depend on the wavelet scale, thus so-called cone of influence should be estimated for more accuracy [12].

3 Modal parameter estimation

Consider a linear damped multi-degree-of-freedom structure superimposed by N-modes, a response solution of the structure due to ambient external excitation as Gaussian distributed broad-band white noises can be expressed as follows:

\[ X(t) = \sum_{i=1}^{N} A_i \exp(-2\pi \zeta_i f_i t) \cos(2\pi f_{di} t + \theta_i) + X_p \]  

where N: number of combined modes; i: index of mode; \( A_i \): amplitude of i-th mode; \( \theta_i \): phase angle; \( f_i, \zeta_i \): undamped frequency and damping ratio of i-th mode; \( f_{di} = f_i \sqrt{1 - \zeta_i^2} \): damped natural frequency; \( X_p \): perturbation due to noises and effect of external excitation.

There is no convincing study on the effect of the ambient loading on accuracy of the output-only system identification methods. It is noted that some authors (e.g. [2, 5]) used the random decrement technique to eliminate the effect of external excitation and estimate impulse responses of structure. However, this technique as conditional correlation function and averaging processing can weaken high spectral components, but low energies in the signal. Normally, effect of perturbation due to noise and external excitation is eliminated.

Substituting Eqs.(9), (6a) into Eq.(1), one can obtain the wavelet transform coefficient:

\[ W_{\theta}^{X}(\tau, s) = \frac{\sqrt{s}}{2} \sum_{i=1}^{N} A_i \exp(-2\pi \zeta_i f_i \tau) \cdot \exp(-\pi^2 f_b(f_i - f_c)^2) \exp(j(2\pi f_{di} \tau + \theta_i)) \]  

Because the wavelet coefficient is localized at certain fixed scale \( s=s_0 \), thus only i-th mode associated with the wavelet scale \( s_i \), dominantly contributes to Eq.(10), other modes can be negligible. Noting that from Eq.(4) we have \( s_i = f_i / f_c \) or \( s_i f_i - f_c = 0 \), thus the term in Eq.(10) \( \exp(-\pi^2 f_b(s_i f_i - f_c)^2) = 1 \). The wavelet coefficient at scale \( s_i \) can be rewritten as single-degree-of-freedom system at i-th mode:

\[ W_{\theta}^{X}(\tau, s_i) = \frac{\sqrt{s_i}}{2} A_i \exp(-2\pi \zeta_i f_i \tau) \exp(j(2\pi f_{di} \tau + \theta_i)) \]
Substituting time $t$ for translation $\tau$, and expressing Eq.(11) in a form of the Hilbert transform’s analytic signal of instantaneous amplitude and instantaneous phase as follows:

$$W_{x}^{y}(t,s_{i}) = B_{i}(t)\exp(j\varphi_{i}(t)) \tag{12}$$

where $B_{i}(t), \varphi_{i}(t)$: instantaneous amplitude and phase, which are determined as:

$$B_{i}(t) = \sqrt{\frac{s_{i}}{2}} A_{i} \exp(-2\pi\zeta_{i} f_{i} t) \tag{13a}$$

$$\varphi_{i}(t) = 2\pi f_{i} t + \theta_{i} \tag{13b}$$

Logarithmic expression of the instantaneous amplitude, then differentiating logarithmic amplitude, and differentiating the phase angle, one obtains:

$$\frac{d\ln B_{i}(t)}{dt} = -2\pi \zeta_{i} f_{i} \tag{14a}$$

$$\frac{d\varphi_{i}(t)}{dt} = 2\pi f_{i} \sqrt{1 - \zeta_{i}^{2}} \tag{14b}$$

From Eqs.(14a), (14b), the i-th natural frequency and the i-th damping ratio can be estimated as follows:

$$f_{i} = \frac{1}{2\pi} \sqrt{\left(\frac{d\ln B_{i}(t)}{dt}\right)^{2} + \left(\frac{d\varphi_{i}(t)}{dt}\right)^{2}} \tag{15a}$$

$$\zeta_{i} = \frac{1}{2\pi f_{i}} \frac{d\ln B_{i}(t)}{dt} \tag{15b}$$

For estimating damping ratios from the wavelet logarithmic amplitude envelope, the linear fitting technique can be applied. Above-discussed wavelet transform-based procedure has been applied for estimating the natural frequencies and the damping ratios of ambient vibrated data of the structure.

4 Refinement techniques for high-order modes

As above-mentioned, the refinement techniques for the frequency resolution analysis at computed frequency bandwidths are required for estimating modal parameters of the low-energy high-order structural modes. In order to facilitate for estimating the modal parameters of high-order modes with low energy in the measured responses, it is proposed some refinement techniques for analysis and adjustment of the frequency resolution. Principally, analyzing frequency bands are divided into finite smaller bandwidths and frequency bands can be filtered at these bandwidths, then time-frequency resolution is changed depending on each analyzing frequency bandwidth thanks to change of wavelet parameters. Some refinement techniques are presented hereafter.

4.1 Broadband and narrowband filtering

Three simple ways to refine on analysis of the time-frequency resolution at high frequency bands are proposed as follows:

1. Bandwidth resolution: Entire frequency band is divided into several frequency bandwidths, and one applies the modified Morlet wavelet with certain parameters ($f_{c}, f_{b}$) to each frequency bandwidth.

2. Broadband filtering: The response time series is broadband filtered into several bandwidths. This process remains desired frequency band for the wavelet transform coefficients, other undesired bands
can be negligible. Next, the modified Morlet wavelet with adjusted frequency resolution applies to band-passed components.

(3) Narrowband filtering: Similar to the second technique, the signal is filtered not on broad bandwidths, but on narrow bandwidths, which are close around each natural frequency. This technique is much more localized around natural frequencies, but it requires prior information on these natural frequencies on the signal. Also, this technique eliminates effects of undesired frequencies on the wavelet coefficient analysis.

4.2 Discrete wavelet decomposition with multi-resolution analysis

In the continuous wavelet transform, response signal is sampled continuously on all over time-frequency plane and computed at every frequency, thus it creates a redundant information, consumes a lot of time. The time and frequency parameters should be discretized in a mutual dependence and with respect to the Nyquist’s sampling rule. If a binary logarithmic discretization (or dyadic grid) is used $s = 2^m$, $\tau = n2^m$ (m, n: integer indexes denote to scale and wavelet location), the continuous wavelet transform is carried out in this dyadic grid of the time-scale plane, the wavelet function is expressed:

$$\psi_{mn}(t) = 2^{-m/2}\psi(2^{-m}t - n)$$

(16)

If selected wavelets satisfy orthogonal basis $\int_{-\infty}^{\infty} \psi_d(t)\psi_{mn}(t)dt = \delta_{kd}\delta_{kn}$, the time series response $x(t)$ can be decomposed from the convolution operation Eq.(1) as follows [1]:

$$X(t) = \sum_{m=-\infty}^{\infty} \sum_{n=-\infty}^{\infty} W^r(m,n)\psi'^{r}_{mn}(t) = \sum_{m=-\infty}^{\infty} \sum_{n=-\infty}^{\infty} \langle x, \psi^{r}_{mn}\rangle \psi'^{r}_{mn}(t)$$

(17)

The discrete wavelet transform implements the so-called multi-resolution analysis for the wavelet decomposition of response signal. This process of multi-resolution analysis works as digital filters, in which the signal passed through low-pass filters to decompose in low-frequency components and high-pass filters to analyze in high-frequency ones. Thus, the discrete wavelet transform uses both wavelet function $\psi_{mn}(t)$ for high-pass filtering and associated scaling function $\phi_{mn}(t)$ for low-pass one. The scaling function is expressed by the same way as the wavelet function as expressed in Eq.(16), furthermore the scaling functions are also orthogonal. With the wavelet and scaling functions, the response signals can be represented by two summations of wavelet decomposition at certain level m0 [1]:

$$X(t) = \sum_{n=-\infty}^{\infty} \langle X, \phi_{m0,n}\rangle \phi'_{m0,n}(t) + \sum_{m=m0}^{\infty} \sum_{n=-\infty}^{\infty} \langle X, \psi_{mn}\rangle \psi'_{mn}(t)$$

(18a)

$$X(t) = \sum_{n=-\infty}^{\infty} a_{m0}[n]\phi_{m0,n}(t) + \sum_{m=m0}^{\infty} \sum_{n=-\infty}^{\infty} d_{m}[n] \psi_{mn}(t)$$

(18b)

As the result in Eq.(18b), the first summation refers to the low resolution or coarse approximation of $X(t)$ at the level m0 that corresponds to the low-frequency band components, whereas the second one is sum of high-resolution details of $X(t)$, corresponds to the high-frequency band ones. The discrete wavelet decomposition at any level M can be expressed by the summation of approximation coefficient (low-frequency discrete wavelet decomposition in low frequency resolution) at final level M ($A_{M}$) and sum of detail coefficients (higher-frequency discrete wavelet decomposition in higher frequency resolution) at lower level $m \leq M$ ($D_m$) as follows:

$$X(t) = A_M + \sum_{m=1}^{M} D_m$$

(19)
where $A_M$: approximation coefficient at scale level $M$; $D_m$: detail coefficients at lower level $m$. Further reading for the discrete wavelet transform should refer to [1].

### 4.3 Empirical mode decomposition

Recently, the empirical mode decomposition has proposed in [4] to determine empirically so-called intrinsic mode functions from the measured time series, in which the signal can be decomposed into a set of almost orthogonal mono-components in the time domain. The mono-component is conditional to obtain analytic time series with instantaneous amplitude, phase and instantaneous frequency. Algorithm of the empirical mode decomposition to obtain the intrinsic mode functions from measured time series can be found out somewhere (e.g. [4, 8]). As a result, measured response can be decomposed into sum of the intrinsic mode functions and residue (constant or trend) as follows [4]:

$$X(t) = \sum_{i=1}^{N} IMF_i + R_N$$

where $IMF_i$: $i$-th intrinsic mode function; $R_N$: residue.

Practically, only first few low-order intrinsic mode functions are meaningful due to containing of actual natural frequencies, other higher-order functions are pseudo-components which contain pseudo low frequencies. Thus, elimination of the higher-order intrinsic mode function from the response time series does the same as noise filtering.

### 5 Full-scale ambient measurements of five-storey building

Ambient vibration measurements have been carried out on a 5-storey steel structure at Disaster Prevention Research Institute (DPRI), Kyoto University (see Figure 2). Ambient data were recorded at all 5 floor levels and ground as reference, by tri-axial velocity sensors with output velocity signals (VCT Corp., Models UP255S/UP252) with A/D converter, amplifier and laptop computer. All data were sampled for period of 30 minutes per floor (5 minutes per a set-up) with sampling frequency of 100Hz [3]. Sensors arrangement also is indicated in Figure 2.

![Figure 2: 5-storey steel structure and sensors arrangement](image)

Because dynamic behavior of the steel structure is sensitive in X direction, thus only outputs sensors and modal parameters in the X direction have been solved and discussed in this paper. All output sensors were velocity time series, moreover, a single integration in the time domain using a trapezoic integration
approach has been required to obtain output displacements for convenient use. A drift and unknown initial condition of displacements during the time integration have been treated.

Figure 3: Integrated displacements at 5-floor levels in X direction
Integrated displacements at all 5 floors and ground are shown in Figure 3.

6 Results and discussions

Figure 4 shows the wavelet transform coefficient of the displacement data at X_1 on the total frequency band 0÷20Hz and the time duration 50÷150seconds, with the central frequency \( f_c = 2 \) and the bandwidth parameter \( f_b = 20 \). As can be seen from Figure 4, there are only two frequency peaks observed at 1.73Hz and 5.34Hz, which correspond to first two structural modes. Other frequencies of higher-order modes (the 3rd mode, 4th mode and 5th mode) can be estimated. Reason is given here because single frequency resolution has been applied to whole frequency band 0÷20Hz, which is good for low frequency analysis, but not appropriate to higher frequency bands.

Figure 4: Wavelet transform coefficient ox \( X_1 \) at \( f_c = 2, f_b = 20 \)

Refinement techniques have been applied here to detect and estimate higher-order modal parameters. Only analysis form output displacement at floor 1 (\( X_1 \)) is illustrated here for sake of brevity. Figure 5 shows the
wavelet transform coefficients at four frequency bandwidths based on the bandwidth resolution technique. Here, there are bandwidths of total 0÷20Hz such as 0÷5Hz, 5÷10Hz, 12÷16Hz, and 16÷20Hz. The wavelet parameters are changeable corresponding to the frequency bandwidths. Obviously, all the frequency peaks can be observed at each analyzing bandwidths, even close frequencies can be determined in the bandwidth of 12-16Hz.

Broadband and narrowband components from the original data are shown in Figure 6, which frequency bandwidths also are indicated in each filtered components. Concretely, the broadband filtered components of entire band 0÷20Hz are 0÷3Hz, 3÷6Hz, 6÷12Hz, and 12÷24Hz used, whereas narrowband filtering around known natural frequencies are implemented at 1.71÷1.77Hz; 5.32÷5.38Hz; 8.81÷8.87Hz; 13.65÷13.71Hz and 18.1÷18.16Hz. The wavelet transform coefficients of the broadband components and the narrowband components, as shown in Figure 6 are presented in Figure 7 and Figure 8. As can be seen from the Figures 7 and 8, the natural frequencies of the higher-order modes can be clearly and visibly observed in the broadband and the narrowband filtering than the bandwidth resolution without filtering as in Figure 5.

![Figure 5: Wavelet transform coefficient with bandwidth resolution](image)

![Figure 6: Broadband and narrowband filtering components](image)
Furthermore, the broadband filtering and narrowband filtering components require lower frequency resolution than the bandwidth resolution technique to detect and extract the higher-order modes. The wavelet transform coefficients are very clear observed when one applies the narrowband filtering (see Figure 8), especially they are localized in the frequency domain but stretched in the time domain. This characteristic is facilitated for damping estimation.

In next step, the refinement techniques using the discrete wavelet decomposition and the empirical mode decomposition have been applied. Figure 9 shows the discrete wavelet coefficients of the displacement $X_1(t)$ at scale level 8 using Daubechies family wavelet D5.
analysis and possible maximum frequency, frequency bandwidths of the discrete wavelet coefficients of \( D_1 \sim D_8 \) and \( A_8 \) are theoretically bands of 25÷50Hz (\( D_1 \)), 12.5÷25Hz (\( D_2 \)), 6.25÷12.5Hz (\( D_3 \)), 3.12÷6.25Hz (\( D_4 \)), 1.56÷3.12Hz (\( D_5 \)), 0.78÷1.56Hz (\( D_6 \)), 0.39÷0.78Hz (\( D_7 \)), 0.19÷0.39 (\( D_8 \)) and 0÷0.19Hz (\( A_8 \)), respectively. Intrinsic mode functions at level 9 from the empirical mode decomposition of the output displacement \( X_1(t) \) are indicated in Figure 10.

**Figure 9:** Discrete wavelet coefficients at scale level 8 with Daubechies wavelet D5

a.\( D_6 \)/\( D_8 \) and \( A_8 \)

b.\( D_1 \)/\( D_3 \)

**Figure 10:** Intrinsic mode functions from empirical mode decomposition

a.\( IMF_1 \)/\( IMF_4 \)

b.\( IMF_2 \)/\( IMF_9 \)

**Figure 11:** Power spectral densities of discrete wavelet coefficients and intrinsic mode functions
Figure 12 shows the wavelet logarithmic amplitude envelopes of the wavelet transform coefficients of the narrowband components. The logarithmic decrements can be estimated via the linear least-square fitting technique as red lines shown in the same plots, the damping ratios then are determined. However, the questionable point here is that how is the time duration in the wavelet logarithmic amplitude envelopes selected for estimating the damping ratios. There is no any discussion on this matter, but it is generally agreed that the selection of time duration much influences on accuracy and reliability of identified damping ratios. It argues that some following guidelines should be taken into consideration for damping estimation: (1) Time duration should start around clear and maximum peak of wavelet transform coefficient, and (2) Short localized time duration should be preferable.

![Wavelet logarithmic amplitude envelopes](image1)

![Wavelet logarithmic amplitude envelopes](image2)

![Wavelet logarithmic amplitude envelopes](image3)

![Wavelet logarithmic amplitude envelopes](image4)

Figure 12: Damping estimation from wavelet logarithmic amplitude envelopes

<table>
<thead>
<tr>
<th>Techniques</th>
<th>Mode1</th>
<th>Mode2</th>
<th>Mode3</th>
<th>Mode4</th>
<th>Mode5</th>
</tr>
</thead>
<tbody>
<tr>
<td>Power spectral density</td>
<td>1.74</td>
<td>5.35</td>
<td>8.84</td>
<td>13.68</td>
<td>18.13</td>
</tr>
<tr>
<td>Bandwidth resolution</td>
<td>1.74</td>
<td>5.32</td>
<td>8.81</td>
<td>13.64</td>
<td>18.07</td>
</tr>
<tr>
<td>Broadband filtering</td>
<td>1.73</td>
<td>5.34</td>
<td>8.82</td>
<td>13.59</td>
<td>18</td>
</tr>
<tr>
<td>Narrowband filtering</td>
<td>1.73</td>
<td>5.34</td>
<td>8.82</td>
<td>13.59</td>
<td>18</td>
</tr>
</tbody>
</table>

Table 1: Estimated natural frequencies (Hz)

<table>
<thead>
<tr>
<th>Techniques</th>
<th>Mode1</th>
<th>Mode2</th>
<th>Mode3</th>
<th>Mode4</th>
<th>Mode5</th>
</tr>
</thead>
<tbody>
<tr>
<td>Narrowband filtering</td>
<td>0.52</td>
<td>2.07</td>
<td>2.07</td>
<td>1.75</td>
<td>2.22</td>
</tr>
</tbody>
</table>

Table 2: Estimated damping ratios (%)

Results of the natural frequencies and damping ratios extracted from mentioned three techniques are listing this Table 1. There is good agreement observed for the identified natural frequencies between the techniques.
7 Results and discussions

Output system identification of ambient data using the wavelet transform has been presented in the paper. Modified Morlet wavelet is more adaptive and appropriate to treat with sophisticated time-frequency resolution analysis. Some frequency resolution techniques have been proposed to estimate the modal parameters of the high-order modes and the close frequency problem. Identified natural frequencies from three techniques are good agreement. However, further investigations on selection of time duration in the wavelet logarithmic amplitude envelopes are required for more reliability and accuracy of the damping estimation.

Acknowledgements

This study was funded by the Ministry of Education, Culture, Sport, Science and Technology (MEXT), Japan through the Global Center of Excellence (Global COE) Program, 2008-2012. Ambient data have been provided for use by Dr. Kuroiwa from Structural Dynamics Laboratory, Kyoto University, Japan for which the first author would express the many thanks.

References

